

# CHAPTER 3

## LAYING YOUR FOUNDATION

### *It Starts With Big Ideas, Essential Questions, and Standards*

As the science and mathematics teacher on my team, I am responsible for practical math, prealgebra, and Algebra I along with my science classes. At lunch the other day, I was sitting with my math and science colleagues. I mentioned to Mo, another science teacher, that I had just come up with a great essential question for our unit on matter. “How about the question, ‘Does matter behave predictably?’” Mo responded that she liked it. Then Jamel, a fellow prealgebra teacher, piped in, “I didn’t know you had essential questions in science, too!”

“What do you mean by ‘too’? You do them in math?” I exclaimed.

“We sure do,” said Jamel. “Don’t you do them for your math classes? It sounds like you are really good at them!”

This conversation at lunch really had me thinking. I don’t use essential questions in math. But why not? They guide my science lessons, and I always share them with my students so that we know why we are studying and doing the labs we are doing. Why did I never make that connection to math? My lessons always go fine, but I think about math skills as being needed to do the next step in mathematics. The skills build on one another. It never occurred to me that there is an overarching purpose to the mathematics skills. If I had essential

questions in mathematics, it would sure help me tie the lessons together. I am going to try this!

Kim McCormick  
Middle School Mathematics and Science Teacher  
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**Many teachers think about mathematics as skills, like Kim, or simply find it difficult to write essential questions for mathematics. In this chapter, we will focus on big ideas, essential questions, and standards as the building blocks of a lesson taught at the 6–8 grade levels. We will also address the following questions:**

- What are state standards for mathematics?
- What are essential questions?
- What are process standards?

## WHAT ARE STATE STANDARDS FOR MATHEMATICS?

For many years, research studies of mathematics education concluded that to improve mathematics achievement in the United States, standards needed to become more focused and coherent. The development of common mathematics **standards** began with research-affirmed **learning progressions** highlighting what is known about how students develop mathematical knowledge, skills, and understanding. The resulting document became known as the *Common Core State Standards for Mathematics (CCSS-M)* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The landmark document was intended to be a set of shared goals and expectations for the knowledge and skills students need in mathematics at each grade level. The overall goal was college and career readiness.

Currently, the majority of states have adopted the *Common Core State Standards for Mathematics* as their own state standards. However, it is important to note that while many states adopted the CCSS-M, others have updated, clarified, or otherwise modified them, adopting the updated set as their new state standards. A few states have written their own standards.

Most standards documents are composed of **content standards** and **process standards** of some kind. It is important to recognize that no state standards describe or recommend what works for all students. Classroom teachers, not the standards, are the key to improving student learning in mathematics. The success of standards depends on teachers knowing how to expertly implement them. It is important as a teacher to be very knowledgeable about your own state standards and what they mean, not only at your grade level but also at the one above and below the one you teach. They are at the heart of planning lessons that are engaging, purposeful, coherent, and rigorous.

Regardless of whether your state has adopted CCSS-M, has modified the standards, or has written their own, the **big ideas** of 6–8 mathematics are universal. Big ideas are statements that describe concepts that transcend grade levels. Big ideas provide focus on specific content. Here are the big ideas for algebraic thinking in Grades 6 through 8.

### Sixth Grade

The study of algebraic thinking focuses on variable expressions and equations. Students write and evaluate numerical expressions. Sixth graders also solve simple one-step equations such as  $4x = y$  to describe relationships. This study of expressions and equations lays the foundation for the transition to algebraic representation and problem solving in later grades.

### Seventh Grade

Properties of operations in Grade 7 are used to generate equivalent expressions. Students use the arithmetic of rational numbers to create expressions and equations in one variable and use these equations to solve problems. The focus on solving real-world and mathematical problems using algebraic equations lays the groundwork for eighth-grade equation work and contributes to the understanding for writing nonlinear expressions and equations in later grades.

### Eighth Grade

Eighth grade builds on previous experiences and focuses on more complex equations by learning about and applying the properties of integer exponents, square and cube roots, and scientific notation. Students connect previous understandings about proportional relationships to linear equations. Systems of two linear equations in two variables are introduced with three solution strategies.



**While the major topics in 6–8 algebraic thinking are the same, states may have a slightly different focus at a grade level. Compare your state standards for your grade level with the summary previously stated. Are there any differences? If so, what are they? Briefly describe them here.**

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## WHAT ARE ESSENTIAL QUESTIONS?

It is estimated that over the course of a career, a teacher can ask more than two million questions (Vogler, 2008). If teachers are already asking so many questions, why do we need to consider essential questions? An **essential question** is a building block for designing a good lesson. It is the thread that unifies all of the lessons on a given topic to bring the coherence and purpose discussed previously. Essential questions are purposefully linked to the big idea to frame student inquiry, promote critical thinking, and assist in learning transfer. (See Chapter 5 for more information on essential questions in transfer lessons.) As a teacher, you will revisit your essential question(s) throughout your unit.

Essential questions include some of these characteristics:

- *Open-ended.* These questions usually have multiple acceptable responses.
- *Engaging.* These questions ignite lively discussion and debate and may raise additional questions.
- *High cognitive demand.* These questions require students to infer, evaluate, defend, justify, and/or predict.
- *Recurring.* These questions are revisited throughout the unit, school year, other disciplines, and/or a person's lifetime.
- *Foundational.* These questions can serve as the heart of the content, such as a basic question that is required to understand content to follow.

Not all essential questions need to have all of the characteristics. Here are some examples of essential questions for Grades 6 to 8.

- How can equations and inequalities be used to solve, model, and/or analyze mathematical situations?
- How can geometric properties be used to describe, model, and analyze situations?
- How are the situations described by equations different from the situations described by inequalities?
- How does the type of data collected affect the choice of graph used to display data?
- How do we use probability and data analysis to make predictions?
- Where are integers used in the real world?



**Look at the list of sample 6–8 essential questions. Decide which characteristics describe which question. Note any thoughts or comments below.**

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## WHAT ARE PROCESS STANDARDS?

Up to this point, we have been discussing content standards. However, every state also has a set of standards that define the **habits of mind** students should develop through mathematics. In 1989, the National Council of Teachers of Mathematics (NCTM) introduced these standards as process standards, stating that “what we teach [in mathematics] is as important as how we teach it” (NCTM, 1991), encouraging us to teach mathematics through these processes. Those standards are the following:

- 1. Problem Solving:** Students use a repertoire of skills and strategies for solving a variety of problems. They recognize and create problems from real-world situations within and outside mathematics to find solutions.
- 2. Communication:** Students use mathematical language, including terminology and symbols, to express ideas precisely. Students represent, discuss, read, write, and listen to mathematics.
- 3. Reasoning and Proof:** Students apply inductive and deductive reasoning skills to make, test, and evaluate statements to justify steps in mathematical procedures. Students use logical reasoning to analyze and determine valid conclusions.
- 4. Connections:** Students relate concepts and procedures from different topics in mathematics to one another and make connections between topics in mathematics and other disciplines.
- 5. Representations:** Students use a variety of representations, including graphical, numerical, algebraic, verbal, and physical, to represent, describe, and generalize. They recognize representation as both a process and a product.

The Common Core State Standards have the eight **Standards for Mathematical Practice** (SMPs), which also describe the habits of mind students should develop as they do mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The following SMPs are the same across all grade levels.

- 1. Make sense of problems and persevere in solving them.** Students work to understand the information given in a problem and the question that is asked. Students in Grades 6 to 8 formulate equations and inequalities to model a given situation using information in the problem and use a strategy to find a solution. They check to make sure their answer makes sense.
- 2. Reason abstractly and quantitatively.** Students make sense of quantities and their relationships in problem situations. They use real numbers and variables in mathematical expressions, equations, and inequalities to represent a wide variety of real-world contexts. They examine patterns in data and assess the degree of linearity of functions. Quantitative reasoning involves creating a representation of a problem, understanding the units involved, and attending to the meaning of the quantities, not just how to compute them.
- 3. Construct viable arguments and critique the reasoning of others.** Students make conjectures and build a logical progression of statements to support their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. They construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, tables, and other data displays.
- 4. Model with mathematics.** Students use representations, models, and symbols to connect conceptual understanding to skills and applications. Students can apply the mathematics they know to solve problems in their everyday lives. They analyze relationships mathematically to draw conclusions and interpret mathematical results in the context of the situation and decide if the results make sense.
- 5. Use appropriate tools strategically.** Students use tools when solving a mathematical problem. These tools may include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software.
- 6. Attend to precision.** Students learn to communicate precisely with each other and explain their thinking using appropriate mathematical vocabulary. They also calculate accurately and efficiently and express numerical answers with a degree of precision.

7. **Look for and make use of structure.** Students discover patterns and structure in their mathematics work and use it to their advantage when solving problems. They can also articulate what they discover to share with their peers and teachers.
8. **Look for and express regularity in repeated reasoning.** Learners notice repeated calculations and begin to make generalizations. By recognizing what happens when using positive and negative exponents, students extend that understanding to more difficult problems. This standard mentions short-cuts. However, short-cuts are only appropriate when students discover them by making generalizations on their own and understand why they work.

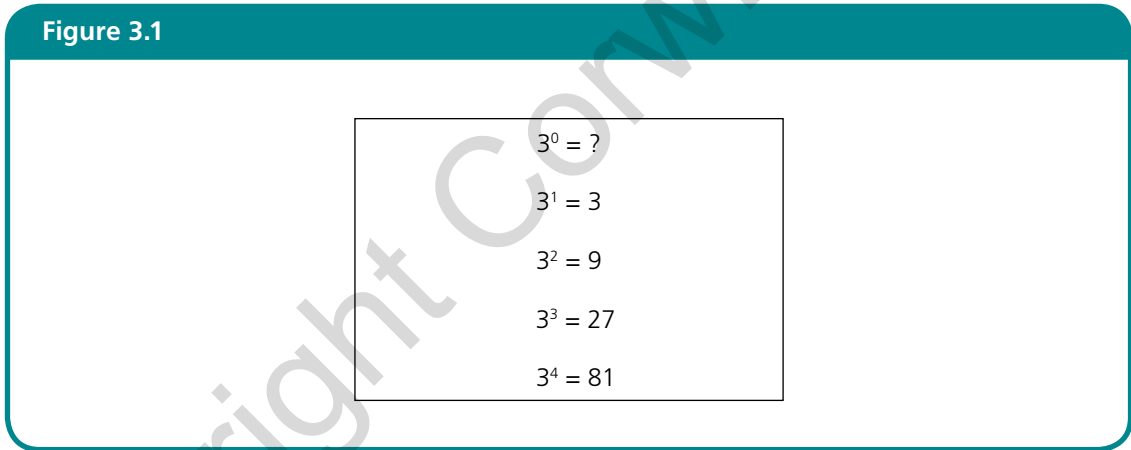
The SMPs are not intended to be taught in isolation. Instead, you should integrate them into daily lessons because they are fundamental to thinking and developing mathematical understanding. As you plan lessons, determine how students use the practices in learning and doing mathematics.

Both sets of standards overlap in the habits of mind that mathematics educators need to develop in their students. These processes describe practices that are important in mathematics. Not every practice is evident in every lesson. Some lessons/topics lend themselves to certain practices better than others. For instance, you might use **classroom discourse** to teach a content standard through important mathematical practices.

**Example:** Manuel

Manuel, an eighth-grade teacher, creates a problem using exponents to have his students engage in constructing viable arguments and critiquing the reasoning of the others. He displays the following (see Figure 3.1):

**Figure 3.1**



Manuel then asks his students the following: “Using what you see, figure out what  $3^0$  is and justify your thinking.” Students work in pairs to analyze the display. After a few minutes, Manuel calls the class back together for a discussion.

**Manuel:** So who would like to tell us something they discovered?

**Betty:** Well, the exponents are all in order, 0, 1, 2, 3, 4, but I never saw a zero exponent before.

**Jimmy:** The answers go up by multiples of 3.

**Shaq:** Those aren’t multiples. You would have a 6 in there. But they do go up by multiplying the answers by 3.  $3 \times 3 = 9$ ,  $9 \times 3 = 27$ ,  $27 \times 3$  is 81.


**Janine:** It works backward, too.  $\frac{81}{3}$  is 27.  $\frac{27}{3}$  is 9.  $\frac{9}{3}$  is 3.

**Betty:** So  $3^0$  must be 0 because  $3^1 = 3$ , and if you take away another 3, you get 0.

**Manuel:** Does anyone else agree with Betty's thinking?

**Shaq:** I don't think I do. We are not subtracting; we are dividing. If you keep going backward like Janine said with division, you go  $\frac{9}{3} = 3$ , and  $\frac{3}{3} = 1$ . So  $3^0$  must be 1.

Through classroom discourse, Manuel asked a carefully selected question to have his students engage in constructing viable arguments and critiquing the reasoning of the others. He did not point out his students' misconceptions. He let them critique each other's reasoning. This is an example of how content can be taught through important mathematical practices.



**Think about the process standards/mathematical practices included in your state standards. Select one and reflect on how you weave it into your lessons.**

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It is important to note that the decision to start with a big idea, essential question, or standard is up to you. Some school systems have  **pacing guides**  and  **district-wide curricula** , which dictate the order in which the standards must be taught. In that case, you need to do the following:

- Look at your standards and decide which big ideas it covers.
- Identify the common thread or essential question you want to weave through your lessons on this big idea.

If your district does not have a pacing guide, you may first want to select a big idea to teach and then select the state standards you will cover in the lessons.



## Building Unit Coherence

One of the best ways to build coherence between and among lessons within your unit is through the big ideas, essential questions, and standards. Keep in mind that connecting individual lessons through these three main elements promotes in-depth conceptual understanding, supports coherence, and unifies individual lessons. In fact, your lessons will share big ideas, essential questions, and shared standards within one unit. A big part of creating a coherent unit is strategically deciding how these three elements will be connected. Consider mapping the three components for the entire unit as you develop the lesson plan (Figure 3.2).

Figure 3.2

Unit-Planning Template		
Unit Topic:		
Unit Standards	Unit Big Ideas	Unit Essential Questions



Download the Unit-Planning Template from [resources.corwin.com/mathlessonplanning/6-8](https://resources.corwin.com/mathlessonplanning/6-8)



### Sixth-Grade Snapshot

## Big Ideas, Essential Questions, and Standards

The sixth-grade team, Josh and Jeff, are developing lessons to teach algebraic expressions. After discussing the ups and downs of last year's instruction, they decided to write an essential question to help focus the lesson. Josh said, "You know, Jeff, I think we may have our answer for an essential question. Remember how the students are always asking, 'When are we ever going to use this?' Why not focus our lessons on real-world situations so that our students will be able to answer their own question?" Jeff replied, "What a great idea!"

<p><b>Big Idea(s):</b></p> <p>Arithmetic concepts extend to understanding of algebraic expressions and equations.</p>	<p><b>Essential Question(s):</b></p> <p>What are some everyday situations that can be expressed as algebraic expressions and equations?</p>
<p><b>Content Standard(s):</b></p> <p>Write, read, and evaluate expressions in which letters stand for numbers.</p>	<p><b>Mathematical Practice or Process Standards:</b></p> <p>Reason abstractly and quantitatively. Attend to precision. Look for and make use of structure.</p>

See the complete lesson plan in Appendix A on page 188.



What kinds of essential questions can you ask that encompass big ideas in your class? Record some of your responses below.

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Standards

LI and SC

Purpose

Tasks

Materials

Student Thinking

Lesson Structures

Form. Assess.

Lesson Launch

Lesson Facilitation

Closure





## Seventh-Grade Snapshot

# Big Ideas, Essential Questions, and Standards

The seventh-grade math team, Alix, Kyle, and Bryan, had been collaboratively planning for almost a year when a new teacher, Kia, joined them. Alix explains, "We like to start by talking about the standard first. We have noticed over the past year that we were interpreting the standard differently. Now, we take a few minutes to talk about the standard and then move onto the big ideas for the standard. This effort has kept us on the same page." Kia responds, "I appreciate that you take the time to talk about what the standard means. I know the math, but it is different when we talk about the big ideas from the standard. And I am interested in hearing everyone else's ideas."

### Big Idea(s):

Students use prior understanding of the four operations with rational numbers and apply the operations in measurement contexts to solve real-world problems.

### Essential Question(s):

Can an expression or equation be written to represent a real-life mathematical problem?

### Content Standard(s):

Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.

### Mathematical Practice or Process Standards:

- Model with mathematics.
- Look for and make use of structure.
- Critique the reasoning of others.

See the complete lesson plan in Appendix A on page 192.



What kinds of essential questions can you ask that encompass big ideas in your class? Record some of your responses below.

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## Eighth-Grade Snapshot

# Big Ideas, Essential Questions, and Standards

Serena is the only person on her team who teaches eighth-grade math. The other core math teacher has Algebra I and geometry classes. Serena finds this difficult when planning because she always likes to bounce ideas around. Right now, she is thinking about essential questions for her graphing unit. Her standards and pacing are determined by her district curriculum and pacing guides. She participated in a workshop this summer on essential questions and remembered that the instructor offered to help anyone in the class via e-mail. Serena immediately sent an e-mail and was pleased to hear back so quickly. After she and her instructor chatted online about the role of graphing in algebra, Serena knew just what she should do.

### Big Idea(s):

Functions can be represented verbally, graphically, symbolically, physically, and in a table.

### Essential Question(s):

What do different shapes of graphed data tell us?

### Content Standard(s):

Understand the connections between proportional relationships, lines, and linear equations.

### Mathematical Practice or Process Standards:

Model with mathematics.  
Look for and make use of structure.

See the complete lesson plan in Appendix A on page 203.



Are there other topics in your grade level that could be guided by an essential question? Give some examples below.

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Standards

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Student Thinking

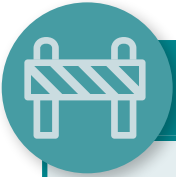
Lesson Structures

Form. Assess.

Lesson Launch

Lesson Facilitation

Closure



## Under Construction

Now it is your turn! You need to decide what big idea, essential question, and standards you want to build a lesson around. Start with your big idea and then identify the remaining elements.

**Big Idea(s):**

**Essential Question(s):**

**Content Standard(s):**

**Mathematical Practice or Process Standards:**



Download the full Lesson-Planning Template from [resources.corwin.com/mathlessonplanning/6-8](https://resources.corwin.com/mathlessonplanning/6-8)

Remember that you can use the online version of the lesson plan template to begin compiling each section into the full template as your lesson plan grows.