STANDARD 1 (6.RP.A.1)

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

In this standard, students learn to compare two quantities or measures such as 6:1 or 10:2. These comparisons are called ratios. Students discover that ratios can be written and described in different ways. For instance, 6:1 uses a colon to separate values. Ratios can also be stated with words such as 6 to 1, or as a fraction such as $\frac{6}{1}$. Standard 1 focuses on understanding the concept of a ratio, however, students should use ratio language to describe real-world experiences and use their understanding for decision making.

What the TEACHER does:

- Help students discover that a ratio is a relationship or comparison of two quantities or measures. Ratios compare two measures of the same types of things such as the number of one color of socks to another color of socks *or* two different things such as the number of squirrels to birds in the park. Ratios compare parts to a whole (part:whole) such as 10 of our 25 students take music lessons. Ratios can also compare a part of one whole to another part of the same whole (part:part) such as the ratio of white socks in the drawer to black socks in the drawer is 4:6. Ratios are expressed or written as *a* to *b*, *a*:*b*, or $\frac{a}{b}$.
- Compare and model ratios with real-world things such as pants to shirts or hot dogs to buns. Ratios can be stated as the comparison of 10 pairs of pants to 18 shirts and can be written as $\frac{10}{8}$, 10 to 18, or 10:18 and simplified to, $\frac{5}{9}$, 5 to 9, or 5:9. Ensure that students understand how the simplified values relate to the original numbers.
- Ask students to create or find simple real-world problems to use in their learning such as, "*There are 2 Thoroughbred horses and 6 Appaloosa horses in the field. As a ratio of Thoroughbreds to Appaloosas it is:* $\frac{2}{6}$ *or 2 to 6 or 2:6 or simplified as* $\frac{1}{3}$*, 1 to 3, or 1:3. Or, there are 14 girls and 18 boys in our math class. As a ratio of girls to boys it is:* $\frac{14}{18}$*, 14 to 18, or 14:18 or simplified as* $\frac{7}{9}$*, 7 to 9, or 7:9.*" Invite students to share their real-world examples of ratios and use ratio language to describe their findings such as, "*for every vote candidate A received, candidate C received nearly three votes.*" The problems students select or write can also be used as cyclical reviews with distributed practice throughout the school year.
- Focus on the following vocabulary terms: *ratio*, *compare*, and *simplify*.

What the STUDENTS do:

- Understand that a ratio is a comparison between quantities.
- Determine when a ratio is describing part-to-part or part-to-whole comparison.
- Describe ratio relationships between two quantities using ratio language.
- Use the different ratio formats interchangeably (4:5, 4 to 5, $\frac{4}{5}$).

Addressing Student Misconceptions and Common Errors

Some sixth graders may confuse the order of the quantities such as when asked to write the ratio of boys to girls in the sentence, "*There are 14 girls and 18 boys in our math class*." Instead of writing 18:14, some students may write 14:18. Other students may not recognize the difference between a part-to-part ratio and a part-to-whole ratio such as, "*There are 14 girls compared to 18 boys in the class (14:18 part-to-part); however, 14 of the 32 students in our class are girls (14:32 part-to-whole)*." To address these common misconceptions, ask students to label the quantities they are comparing such as 14 girls/18 boys.

STANDARD 2 (6.RP.A.2)

Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."[1]

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

This standard focuses student learning on the concept of a unit rate as a special kind of ratio. Students compare different units of measure such as the amount of money earned to the hours worked while babysitting and calculate unit rates by setting up ratios and simplifying them. Students understand a situation in ratio form and write the unit that describes the situation using appropriate rate language with words such as *per* and symbols such as / to compare different units or measures.

What the TEACHER does:

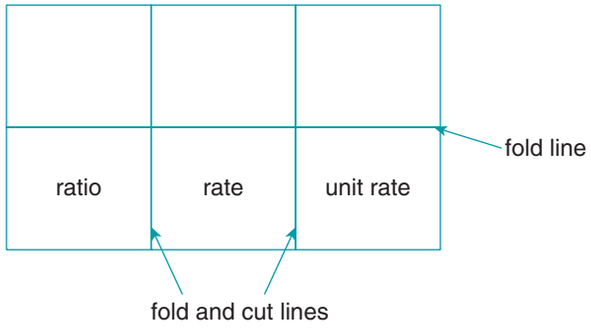
- Begin by exploring the difference between a ratio and a rate. Rate is a special ratio that compares two quantities with different units of measure. Share multiple examples for students to make sense of the concept for rate such as, "*LaShanda babysat for $35 for 7 hours.*" Or, "*Dad's new truck got 400 miles on 20 gallons of gas.*" Then explore the unit rate that expresses a ratio as part-to-one. Generate examples such as "*LaShanda is paid a unit rate of $5 per 1 hour for babysitting (5:1)*" and "*My dad's new truck gets 20 miles per gallon of gas (20:1).*"
- Ask students to locate and share real-world examples of cost per item or distance per time in newspapers, ads, or other media. (Note that in sixth grade, students do not work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.)
- Model how to convert rates from fraction form to word form using per, each, or @ such as 360 miles/12 gallons of gas = 30 miles per gallon of gas. Allow students to talk with each other and their teacher to make sense of what they are learning and then write and share several rate conversion examples of their own.
- Focus on the following vocabulary terms: *ratios*, *rates*, *unit rates*, *compare*, and *per*/@. Math journals or exit slips at the end of a math class with writing prompts such as, "*An example of a ratio and a problem that goes with it is. . . .*" provide closure.
- Provide cyclical, distributed practice over time to continually review simple unit rate problems.

What the STUDENTS do:

- Understand rate as a ratio that compares two quantities with different units of measure.
- Understand that unit rates are the ratio of two measurements or quantities in which the second term means "one" such as 60 miles per one hour.
- Interpret rate language with the @ symbol or with the words *per* and/or *each*.
- Solve unit rate problems.

Addressing Student Misconceptions and Common Errors

Students often confuse the terms *ratio*, *rate*, and *unit rate*. Try using a paper foldable with vocabulary definitions to help students with these confusing terms. To make the foldable, divide an $8\frac{1}{2}$ × 11-inch sheet of blank paper in half horizontally. Then fold it into thirds as if a letter is being folded to fit an envelope. Unfold and write a term on each of the sections. On the inside of the foldable, write the definitions that match each term. Students may want to cut on the vertical fold lines to flip up each section to practice the definitions.



STANDARD 3 (6.RP.A.3)

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. *Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.*

b. *Solve unit rate problems including those involving unit pricing and constant speed.* For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

c. *Find a* ***percent*** *of a quantity as a rate per 100 (e.g., 30% of a quantity means* $\frac{30}{100}$ *times the quantity); solve problems involving finding the whole, given a part and the percent.*

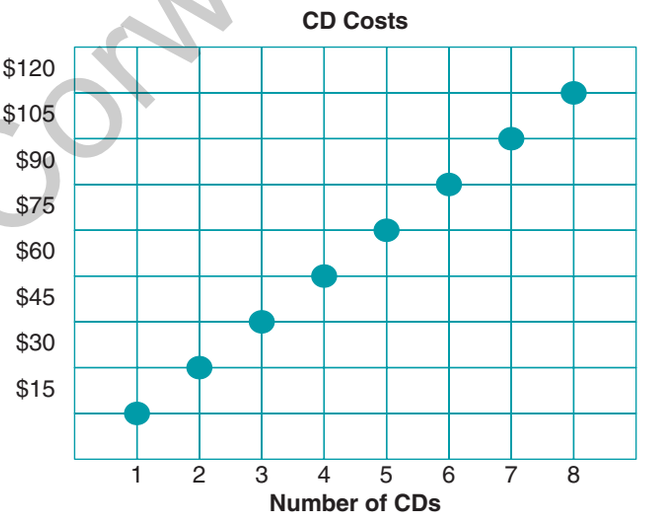
d. *Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.*

In these standards, students use reasoning about multiplication and division to solve a variety of ratio and rate problems about quantities. They make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. They use tables to compare ratios and solve unit rate and constant speed problems. Problems involving finding the whole given a part and the percent, such as 20% of a quantity means $\frac{20}{100}$, are also a focus. For these standards, students can use equivalent ratio tables, tape diagrams, double number lines, or equations. Students connect ratios and fractions.

What the TEACHER does:

- Explore ratios and rates used in ratio tables and graphs to solve problems. Pose a ratio situation problem with students such as "3 *CDs cost $45. What would 8 CDs cost? How many CDs can be purchased for $150.00?*" To solve the problem, students can use ratios, unit rates, and multiplicative reasoning by creating and filling in the missing values on a chart. They should note that if three CDs cost $45, one CD will cost $15. Every CD purchased is an additional $15. $15 times the number of CDs = the cost. They write an equation such as C = $15*n*.

# of CDs	Cost
3	$45
8	??



- Ask students to plot the points on a coordinate plane and draw conclusions about what is happening with the problem above. Students should reason that for every one movement to the right on the *x*-axis, the *y*-axis increases to 15*x*. Also, for every one movement to the left on the *x*-axis, the *y*-axis decreases by 15.

- Investigate unit rate problems, including unit pricing such as, "Quick Stop *has 12-oz. drinks for* $.99. Stop Here *has 16-oz. drinks for $1.19. Which drink costs the least per ounce?*" Assign students to create ratio and rate reasoning examples to compare and solve real-world problems. Students could use newspapers, store ads, or online ads to find the examples and make the comparisons. Ask students to use reasoning to determine the better buys.

- Explore finding a percent of a quantity as a rate per 100 such as 40% of a quantity means $\frac{40}{100}$ times the quantity. Noting that a percent is a rate per 100, model how a

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What the TEACHER does (continued):

percent can be represented with a hundreds grid by coloring in 40 units. Have students write this as a fraction ($\frac{40}{100}$), a decimal (0.40), and a percent (40%). Consider using a percent wheel (see Reproducible 1) or use double number lines and tape diagrams in which the whole is 100 to find the rate per hundred.

- Solve problems involving finding the whole, given a part and the percent such as, "*What is 40% of 60? 80% of what number is 300? Or 50 is 30% of what number?*"
- Examine the process of how to use ratio reasoning to convert measurement units such as, "*How many centimeters are in 5 feet?*" Use the information that 1 inch ≈ 2.54 cm. Represent the conversion of 12 inches = 1 ft as a conversion factor in ratio form, $\frac{12 \text{ inches}}{1 \text{ foot}}$.

$$\text{Then multiply } \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{5 \text{ ft}}{1} = 60 \text{ inches.}$$

$$\text{Then } 60 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 152.4 \text{ cm.}$$

(Note that conversions can be made between units within a measurement system such as inches to feet or between systems such as miles to centimeters.)

- Allow students to talk with each other and their teacher to make sense of what they are learning.
- Focus on the following vocabulary terms: *ratios*, *rates*, *unit rates*, *equivalent ratios*, *percents*, *ratio tables*, and *tape diagrams*.
- Provide cyclical, distributed practice over time to continually practice unit rate problems.

What the STUDENTS do:

- Create and interpret a table of equivalent ratios.
- Plot pairs of values from a table to a coordinate plane.
- Use a table to compare ratios and find missing values using ratios.
- Explain the difference between a ratio and a unit rate.
- Understand that rate problems compare two different units, such as revolutions per minute.
- Solve real-world problems using ratios and rates.
- Reason to determine the better buy.
- Write a percent as a rate over 100, including percents greater than 100 and less than 1.
- Find the percent of a number using rate methods.
- Represent the relationship of part to whole to describe percents using models.
- Convert units by multiplication or division.

Addressing Student Misconceptions and Common Errors

Some sixth graders misunderstand and believe that a percent is always a natural number less than or equal to 100. To help with this misconception, provide examples of percent amounts that are greater than 100% and percent amounts that are less than 1%. Try using a percent wheel for developing this understanding. See Reproducible 1.

Notes