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# How Children Learn Mathematics and the Implications for Teaching

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**Chapter Overview**

In this chapter you can read about:

- Why and how young children learn mathematics
- The importance of practical activities
- Starting with children's interests
- Children solving problems
- The progression of children's mathematical ideas from birth to 8 years old.

**The nature of mathematics and young children learning**

- What do you think mathematics is?
- Is it appropriate and important for very young children?
- At what age do you think children should start learning mathematical ideas?
- How can adults help?

Mathematics is defined as 'the abstract science of number, quantity and space' by *The Concise Oxford Dictionary* (Allen, 1990: 732). It can be seen as a way of organising ideas in order to develop concepts. Skemp (1971) identified that having a concept involves more than knowing its name; it involves being able to use the understanding developed from recognition of similarities between particular examples of the concept. Freudenthal (1973) argued that children develop concepts as a result of experiences and thinking about those experiences. This may seem remote from children aged from birth to 8, however babies and young children are naturally curious and explore their world from an early age. They do not compartmentalise their explorations as mathematical or otherwise. However, they will encounter mathematical ideas such as number, quantity and shape. Mathematics is an essential area of understanding and knowledge in our everyday lives as adults and this is also true for children. The National Curriculum (DfEE, 1999a: 60) states 'Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem solving skills and the ability to think in abstract ways'. Lee (2006) states that mathematics empowers and enables us to take control of various aspects of our lives. It is also a creative discipline, capable of being used flexibly to communicate precisely. Mathematics can also be seen as a web of ideas that are continually refined and developed. According to ACME (2008: 4) the 'big ideas' of mathematics for young children include 'place value and the number system, conservation of number and measures, equivalence relations and dimensionality' (see chapters in Part 2).

Parents and carers will often intuitively draw young children's attention to mathematical ideas by pointing out and talking about numbers, quantities, shapes and sizes as part of everyday life. This will be continued when they sing and recite rhymes to, and with, their children and as they play with their children using a variety of toys and objects (for example, wooden bricks and other construction toys, soft toys, small-world toys and toy versions of real objects like tea sets). Mathematics features in the routines of everyday life, such as getting dressed and putting on two socks or laying the table and getting three plates out. Children begin to use vocabulary that reflects understanding of mathematics such as when they ask for more chocolate, sweets or chips. From these early mathematical experiences and ideas, children will gradually extend their understanding to more formal mathematics.

## How do young children learn mathematics?

A number of theorists have proposed ideas about how children learn generally, and these ideas can be related to the learning of mathematics. Piaget (in Donaldson, 1978) believed that children construct their own knowledge and understanding through their interactions with their environment. This is called a constructivist theory.

Vygotsky (in Atherton, 2011) is often referred to as a social constructivist. He emphasised the need for a child to have guidance from a 'more knowledgeable other' and to have opportunities to interact socially with peers as a means of learning. He also proposed the idea of the 'Zone of Proximal Development', which is that a child can work with someone else to achieve something that they could not achieve on their own, thereby learning through this process so that eventually they are able to perform the task by themselves. This is sometimes called scaffolding (Bruner, 1966; Wood, 1998). Similarly, Gifford (2008) refers to cumulative learning, meaning that learning needs to build on previously learnt ideas and that presenting children with something too advanced will not be effective.

Mathematical learning is associated with the development of mathematical understanding. Barmby et al. (2009) see this as a continuum where children add to and refine previous understandings. This builds on the work of Bruner (1966) who identified the idea of the spiral curriculum, where children meet an idea at one level and then later meet the idea again but are able to study it at a deeper level and achieve a better understanding of it. His influence can be seen in many mathematics curricula documents and in the practice of teaching mathematics. Bruner also suggested that children go through three phases when learning. The enactive phase is when children engage with something concrete in order to explore and manipulate ideas; this could be related to kinaesthetic learning. The second phase, iconic, is when children begin representing the ideas in a more abstract way. This can be supported in mathematics by using models and images so that eventually children can visualise some of them internally to assist their thinking. Finally, children come to the symbolic phase where they can use abstract ideas and ways of representing the mathematics.

Liebeck's (1984) ELPS approach is related to Bruner's enactive, iconic, symbolic phases in some ways. The E stands for Experience – children need practical experience of the ideas to start with. L is for language, and this is where Liebeck's approach differs from Bruner's; Liebeck emphasises the need for children to learn the language of mathematics, highlighting the need for adults and children to talk about the ideas. She then recommends that children go on to represent mathematical ideas through pictures (P) or diagrams before moving on to formal recording of mathematics through the use of symbols (S). Similarly, Gifford (2008) emphasises the importance of multisensory experiences for learning. Froebel (in Beckley, 2011) also emphasised the importance of practical activity for children's learning, including gardening and use of building blocks. This influence can be clearly seen currently in many nursery settings in the UK.

Skemp (1971) described two ways of understanding mathematical ideas that he called 'instrumental' and 'relational understanding'. Instrumental understanding is a shallower form of understanding. For example, we might develop an instrumental understanding of how to add, subtract, multiply and divide using a set procedure or algorithm by memorising the steps required. However, we might not understand how the procedure worked or why the various steps in the procedure are needed. One of the difficulties with this level of understanding is that if our memory of the procedure failed, we would be unable to continue. In contrast, a relational understanding of these procedures would mean that we understand how and why the procedures work.

## The importance of practical activities

The work of Piaget, Bruner and Liebeck all emphasises practical activity as a starting point for learning with young children and Gifford (2008) reports neuroscientific support for this approach, too. Children are naturally curious and explore the world around them. They love to play. Adults working with young children can build on this in order to support mathematical development. One way to do this is to ensure that appropriate toys and other resources are available for children to play with and for the adults to recognise the mathematical potential of these toys and resources. Adults can then observe children interacting with the resources and provide additional resources or play alongside children using appropriate vocabulary and asking appropriate questions to maximise this potential.

- What sort of toys and resources will be helpful in supporting children's mathematical development?
- Do they have to be especially produced for mathematics?
- Which everyday toys that young children play with might also help to develop mathematical ideas?

**CASE STUDY**

### Farm set

Oliver, aged 5, and Daniel, aged 3, both had identical new farm sets. They played together creating fields of different sizes and shapes with the eight fence sections. They created various shapes and sizes and arranged animals in the fields talking about how many animals would fit and whether they wanted big or small animals in the various fields. In each set there was a tree and the boys sometimes included the trees as field boundaries to increase the different shapes they could produce. At one point they used a section of fence to create a common boundary between two fields, leaving two fence sections out. Oliver announced that they had a 'square field and a triangle field'. He went on to talk about the triangle field being smaller than the square field. Then he said that 'triangles are always smaller than squares'. I asked him if this was always true and said I thought I could draw a really big triangle. He rushed off to find pen and paper and drew the largest triangle he could on a sheet of A4 paper, then he cut this out. He then gradually cut into the triangle creating smaller and smaller triangles. He also drew three rectangles, which he called squares on another piece of paper and wrote 'small to [middle] size to big' and told me that he could draw squares in lots of sizes. Taking another piece of paper, he drew a series of five triangles, each one slightly larger than the one before, a similar set of six circles and finally four squares (see Figure 1.1). This demonstrates that he has a good understanding of ordering sizes and that a shape can come in any size, contradicting his previous statement that 'triangles are always smaller than squares'. Daniel, at 3 years old, did not choose to draw anything but was content to talk about the sizes of the animals describing them as big and little and counting a set of four big horses accurately. I attempted to develop his vocabulary by talking to him about 'middle-sized' animals; he appeared to listen but did not use this vocabulary himself.

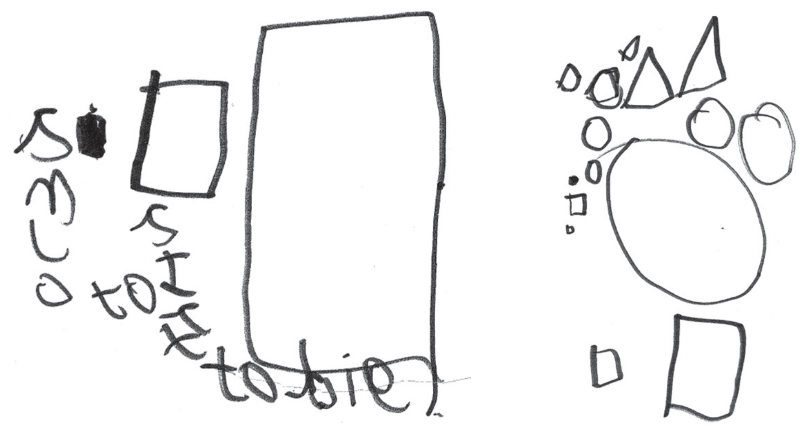

**Figure 1.1** *Oliver's drawings of shapes of different sizes*

## Resources, models and images

The case study is an example of the use of toys that are not made specifically for mathematics. However, various resources have been designed specifically for this purpose. Many of these are designed as models which, together with mathematical

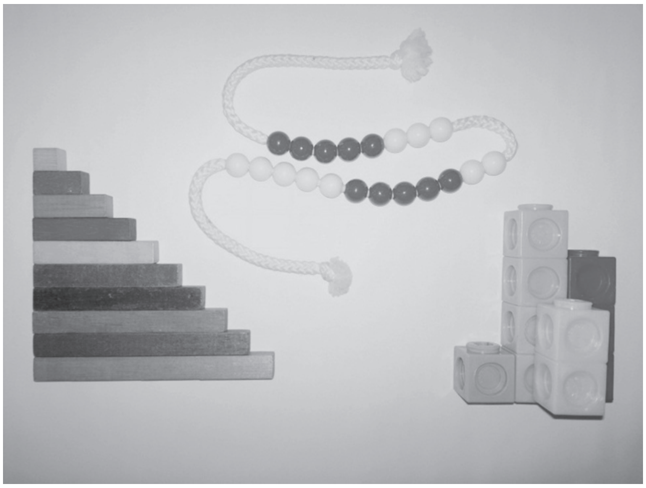

**Figure 1.2** *Bead string, Cuisenaire rods and interlocking cubes*

talk, can support children in building up mental images of mathematical ideas. This type of resource includes objects such as interlocking cubes, Cuisenaire rods and bead strings (see Figure 1.2). Some paper-based resources are also published to support children in building up their own images and mathematical understanding. These include number lines and hundred squares. Delaney (2010) recommends that resources should be used in two different ways. One of these is by a teacher or adult for demonstration of an idea along with explanation. He also recommends that children should engage with resources, directly playing with them, handling them and talking about them so that they can build up their own conceptions of the mathematical ideas.

Higgins (2008) recommends that ICT can be helpful for children in learning about mathematical ideas, if it is carefully selected and accompanied by appropriate talk. Some computer programs can help children in counting and developing early calculation and ideas about shape. Calculators can also be judiciously used to support the development of mathematics, as can interactive whiteboards.

**CASE STUDY**

### Hundred square

Children in a Reception Class aged 4 and 5, had been introduced to the hundred square (see Figure 1.3) by their teacher. Later in the week during a child-initiated session, a group of children chose to draw and make marks on mini whiteboards.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Figure 1.3** *Hundred square*

Harry (aged just 5) had chosen a whiteboard with a 10 x 10 grid marked on it. He fetched a copy of the hundred square that the children had been using previously. He started to write numbers on the whiteboard grid, trying to copy from the hundred square. He wrote 1 and 2 successfully then wrote 3 the wrong way round. He missed out some numbers and wrote some of the teens numbers in the top row. In the square under the number 1, he wrote 100, although he found it difficult to fit the three digits into the small square. I asked him to explain what he was doing. He did not appear to realise that he had missed writing some numbers but he appeared to have a good understanding of how the hundred square worked and how the numbers are traditionally arranged in it. Josh, aged 4 years and 11 months, came over to join in the conversation. He pointed out that the 100 should be in the bottom right-hand square and added 'because that is the end number'. He went on to say, however, that when we're counting 'We can go past one hundred [and] we can go on counting forever because it never stops'. Harry rubbed 100 out from the second row and wrote it in the bottom right-hand square. This demonstrated both children had been supported in their mathematical understanding through the use of the hundred square and that both had a good understanding for their age of the number system and how it works.

## Starting with children's interests

It is important that children develop positive attitudes to mathematics in the early years; therefore they need to be interested and to enjoy mathematical activities and experiences.

Much of the documentation produced by the National Numeracy Strategy (DfEE, 1999b) and the Primary National Strategy (DfES, 2006) in England has recommended that children should be given opportunities to solve 'real-life' problems in mathematics. However, it could be questioned whether a problem within a classroom or early years setting is in fact 'real-life'. It may be preferable to use the term 'meaningful contexts' for problem solving as this can also include fantasy or imaginary contexts, such as stories that are meaningful for children but fictional. In the case study described earlier in this chapter, Oliver and Daniel were engaged in mathematics in a context that was meaningful for them on that particular day as they had just received new farm sets as presents. You will be able to identify other play situations and toys where there is potential for mathematics arising from the children's play. Play and mathematics is the subject of Chapter 3.

Giving children a degree of choice can help them to build up their independence; this in turn will help them to develop persistence and resilience, which are necessary in the learning of mathematics and in problem solving within mathematics, the wider curriculum and beyond. Children need a self-image as a successful learner (Gifford, 2008) and the affective aspect of learning mathematics is extremely important. Adults need to build children's confidence in mathematics as well as their competence.

### Schemas arising from children's interests

Carruthers and Worthington (2006) built on Athey's (1990) work on schemas. These are repeated behaviours during play that demonstrate children's exploration of particular ideas and concepts. Schemas include enclosing things, connecting and transporting. Their research involved asking parents to keep diaries of children's schemas. Adults working with children in various early years settings also took note of children's schemas. Carruthers and Worthington recommend that adults working with young children should work with and extend children's schemas. They give an example of where a child was in a spiral schema, extending this by encouraging her to examine snail shells carefully and to stir fruit into plain yoghurt. They also identified that children may represent ideas associated with their present schema, leading to further exploration of the idea and therefore to deeper learning. They contrast children exploring shapes through their play when they are interested, with a teacher holding up a shape and telling the children that it is a triangle.

- Think of a child in the age range of 0–8 years that you know well. What are their current interests?
- What would be a meaningful context for them?

Routines of the day can also provide meaningful contexts for mathematical development. These could include:

- getting dressed
- laying the table
- tidying up
- preparing a snack.

Often, just by listening and talking to children at these times, mathematical language and vocabulary can be developed. For example, we might talk with children about the sizes of different objects. Sometimes, the situations could be developed into problems for children to solve. Examples of questions we might ask children when laying a table are:

- 'How many plates do we have?'
- 'How many plates do we need?'
- 'How many more plates do we need?'

The first of these is a simple counting question. The second could be more challenging if the people needing the plates are not all present and the child has to count from their imagination. The third question could combine the first two and involve some addition or 'counting on'. Adults can assist children to answer these questions through modelling by thinking aloud.

Stories, songs, poetry and rhymes can also provide meaningful starting points for mathematics. Beyond counting books, some stories have been written especially for the purpose of developing children's mathematical understanding. However, mathematics can also ensue from reading a story that is not directly written with mathematics in mind. An example is *Mr Magnolia* (Blake, 2010). Although it is not obviously a counting book, it works in that way. Mr Magnolia starts the story with one boot, he has two sisters, three creatures in his pond, and so on. There is also scope for discussion of sizes when looking at the pictures. The opportunity to discuss items that come in pairs could also lead to work on odd and even numbers or counting in twos. Giant stories can also lead to discussions of size (see Chapter 13) and perhaps to children solving problems by working out how tall the giant is from a foot or hand print.

Many children will enjoy joining in with songs and rhymes and these can provide a meaningful context from which to explore some mathematical ideas. 'When Goldilocks went to the house of the bears' supports counting to three and talking about different sizes. Very early adding and subtracting can be supported by singing 'Two

Little Dickie Birds'. You can find books (sometimes with a CD) of songs especially written for supporting mathematical development in young children.

## Out of doors and further afield

Young children learn all the time and in different environments. They will be used to this before they join a more formal learning setting such as a nursery, pre-school or school situation. Many children love to play outdoors and much mathematical learning can take place outside, taking advantage of the spaces in the natural and built environment. This could be in an outdoor area adjoining the setting or further away, necessitating a journey. Locations for outdoor learning can be familiar or unfamiliar. Similarly, there are interesting opportunities for learning in different indoor locations. Different locations will appeal to children's interests and can be useful in motivating children to think in different mathematical ways. More consideration will be given to these ideas in Chapter 5.

## Linking mathematical and other learning

Linking mathematical activities within a setting with home and other experiences a child has outside of the setting encourages meaningful learning. This involves getting to know the children well as individuals including their current preferences and interests. Therefore, it is also useful to build productive relationships with the children's parents or carers. Good communication is then far more likely and parents and practitioners working within settings can help children to continue their interests and can talk to the children about them and therefore make learning deeper and more effective (see Chapter 8). Communication between adults within a setting is also very important, especially if a child will come into contact with a number of different adults within a week. Feedback from all adults is also vital if one adult has the key responsibility for the planning of mathematical learning, resources and experiences.

### Links between mathematical ideas

Research into effective teachers of numeracy (Askew et al.,1997) has shown that the most effective teachers of primary mathematics are those who believe that it is important to help children to see the links and connections between different mathematical ideas, different representations of mathematics and children's existing methods and understandings. Teaching is based on dialogue between the teacher and the child and between children to ensure that these connections are highlighted.

Barmby et al. (2009) believe that children develop mathematical understanding through adding mental connections between ideas by representing the mathematics and reasoning. Adults working with young children will often intuitively assist them in making connections, such as helping children to add numbers together by counting. However, if adults are aware of the importance of this, they can capitalise on the opportunities to assist mathematical development. Chapter 10 highlights links between the four operations and it is essential for children to become aware of these.

Links between setting-based and outside experiences can also involve topical themes to be explored in a mathematical context. Young children will not see learning as separated into subject categories and one activity may help their development in several different areas. We have mentioned the development of children's mathematical vocabulary and language several times, because it is bound up with thinking and therefore learning, but this also means that through assisting a child with developing this vocabulary they will be developing their language more generally. Gifford (2008) also highlights this aspect and the way that adults can model mathematical thinking and talk through thinking aloud and helping children to articulate in 'sustained shared thinking' (Siraj-Blatchford et al., 2002).

Figure 1.4 lists some topical themes and related mathematical activities together with the mathematical vocabulary that could be highlighted.

| Theme | Possible mathematics activity | Possible mathematical vocabulary |
|---|---|---|
| Autumn | Collecting, sorting and grouping leaves | Sort, group, the same as, different |
| Christmas | Exploring and creating symmetry in decorating Christmas trees | Symmetrical, match, shape |
| The Olympic Games | Comparing numbers of medals for different countries | More, fewer, the same as, lots, none |
| Holidays | Exploring and counting sand castles | Big, small, size |
| India or Hinduism | Exploring and creating Rangoli type patterns | Pattern, shape, curve, straight |
| National Book Day | Using a story as a starting point for mathematical ideas | Page numbers, order, ordinal numbers, e.g. first, second |
| Bonfire Night | Making and describing a firework picture, counting and describing position and direction | Over, high, up, down, close, far |

**Figure 1.4** *Themes and mathematical activities*

**CASE STUDY**

### Rock pool activity

One summer term I was teaching a class of 4–5 year olds and had chosen the theme of summer holidays to base activities around. I wanted to give the children opportunities to develop positional vocabulary, so I decided to set up the sand or water tray to resemble a rock pool. Stones, pebbles and shells were collected from a beach and washed thoroughly. A large shallow dish of water was placed at one end of the sand tray; this was surrounded by the rocks and pebbles. Sand was then distributed to make a more natural appearance. Children had a collection of sea shells and plastic sea creatures, including crabs and other shellfish, a seahorse and starfish. Groups of children then arranged the sea creatures and shells as they wished in the tray. As they did this an adult talked to the children and discussed where they were placing them. Clipboards and paper were also provided and many of the children drew representations of the rock pool and the creatures, again practising the positional language as they drew and described their pictures. They used phrases such as:

'The crab's under water'

'The starfish is coming out from under the rock'

'I've put the shell on the top of the sandcastle'

I encouraged children to use the phrase 'in between' by modelling sentences for the children, 'The seahorse is in between two rocks'. Some of the children were also later heard using this phrase.

## Children solving problems

Problem solving is important in all areas of learning, but perhaps particularly in mathematics where it provides opportunities for children to develop mathematical thinking skills and to apply their understanding in a meaningful way. It encourages children to draw upon their previous experiences when considering possible ways of solving a particular problem and deciding a course of action. Problem solving features in many of the other chapters in this book, with a particular focus on problem solving within children's play in Chapter 3.

- What constitutes problem solving in mathematics for very young children?

For children to appreciate mathematics as meaningful and relevant to them and their everyday lives and not just a preparation for the future, they will need to be involved in using and applying mathematics to solve problems. Problem solving for very young children should consist mainly of practical problems. It is sometimes easy to think of problem solving in school mathematics being word problems. These would only be relevant for some of the older children within the age group considered by this book, and even then practical problems should be the focus. Problem solving is a key feature of the curriculum guidance from each of the countries of the UK. Children have often been asked to solve some problems by using and applying their recent learning in mathematics lessons. However, young children will often come across problems to solve in their play and daily activities. Sometimes these can lead to the children learning mathematics. Starting with a problem can also be a very effective approach for older children and is regularly used in the Netherlands in 'Realistic Mathematics Education' (van den Heuvel-Panhuizen, 2001).

Children will need to learn problem-solving skills such as:

- identifying the mathematics required
- simplifying
- decision making
- representing
- organising and checking
- recognising patterns
- communicating in different modes
- predicting
- justifying
- explaining
- conjecturing or hypothesising
- generalising.

Some of these will need to be modelled by adults. For example, we might demonstrate to children how to draw a table to organise their results. However, at times children will also need freedom to approach a problem in their own way with opportunities to discuss methods and approaches with others.

Children solve problems of a mathematical nature through their lives and play as and when they occur. Adults may also set up problems for children to solve. Children will sometimes take these ideas forward into their play later on. Figure 1.5 suggests some problem-solving opportunities and gives some examples of problems children may engage with.

*(Continued)*

*(Continued)*

| Problem-solving opportunity | Examples |
|---|---|
| Building with construction toys | Children will often look for a brick or a piece of a certain size to fit a gap. |
| Tidying up | An adult may ask 'Have we got all the scissors?' This might result in a child making sure that each place in the scissor rack has a pair of scissors in it or saying that one pair is missing. |
| Adult suggested problem after a story | After listening to the story of The Three Bears, children might be challenged to use junk modelling materials such as recycled cardboard packaging boxes and pieces of fabric to make beds for teddy bears. This could be supported by talking with the children about whether the various teddies will fit the beds. |

**Figure 1.5** *Problem-solving opportunities*

## Progression in children's mathematical ideas 0–8

As described in Chapter 9, babies have a sense of quantity and as they get older children will gradually build up more experiences and develop their mathematical ideas. Some of this will happen almost naturally and despite what adults do. However, ultimately it is important that children's mathematical progress is maximised. Progress could be seen as cumulative learning (Gifford, 2008). Talk and developing mathematical vocabulary is essential for this maximising of progress (see Chapter 2). Many parents intuitively talk to their children about mathematical ideas and will help their children to progress, for example they will count for and with the child and encourage the child to count further than they have previously. They will use words to describe sizes and praise the child when he or she talks about a big dog.

Once a child joins a more formal setting such as a nursery, pre-school or school, it will be more important for the adults to identify and enable progression; this is more complicated when working with larger numbers of children. Children need new opportunities that build on their previous knowledge and understanding supported by development of their mathematical vocabulary. It is therefore essential that adults understand what children already know, understand and can do in order to provide appropriate challenges, resources and other support for children in their play and in the activities that adults encourage the children to take part in. This understanding is gained through assessment of children's learning (see Chapter 7). However, it is not just adults understanding where children are in their mathematical development that is important. This must feed through to planning the next stages for children's learning. This may be short-term planning in deciding what question to ask the child or what challenge to set immediately, or it may be longer term, in deciding what mathematical opportunities should be presented to the child the next day, next week or next term.

By the time children are 8 years old, they will need to be able to count accurately in steps of different sizes, forwards and backwards. They will have met and begun to understand the four operations of addition, subtraction, multiplication and division. They should have some knowledge and understanding of shape, space and measures. They should also be able to use and apply their mathematical knowledge and understanding in a variety of contexts to solve problems. It is also helpful if children of this age can remember and recall some mathematical facts rapidly, such as number bonds and multiplication tables (see Chapter 10), so they can use them in later mathematics and problem solving.

### Remembering and mathematical learning

Alloway (2006) conducted a literature review into the effects of children's memories on their mathematics. She found that children with poorer working memories struggled with mathematics learning. This would suggest that adults should assist children in this area by giving very limited numbers of instructions at any one time, breaking tasks down into bite-sized pieces and providing memory aids. Children will also need to learn strategies for supporting themselves independently when their memory fails them.

It is vital that young children develop positive attitudes to mathematics in order to prepare them for success in future mathematical development. In order to promote these attitudes children need to be motivated, stimulated, engaged and interested so that they enjoy their mathematical learning.

### Learning through mistakes and misconceptions

#### Using children's misconceptions

Bell's (1993) research, albeit with older pupils, showed that addressing misconceptions through discussion during teaching improves achievement in the short and longer term. Letting pupils initially demonstrate misconceptions was also shown to be more effective than trying to get children to avoid them.

Ryan and Williams (2007), Cockburn and Littler (2008) and Swan (2001) advocate that children can learn effectively through their own and others' mistakes and misconceptions if they are exposed and handled sensitively. Adults can assist this learning by being aware of typical errors that children make in various areas of mathematics, not planning to avoid these but rather to highlight them and use them positively in the learning setting. Building up a positive and supportive ethos and culture (Drews, 2011) is important in the learning setting so that children do not feel humiliated if they make an error but gradually begin to appreciate the value of persistence, resilience and the efforts of other children. Spooner (2002) advocates placing children in situations where they feel in control of identifying errors and misconceptions. Ryan and Williams (2010: 147) recommend that adults working with children take 'a positive view of errors and misconceptions as productive starting points for learning'.

According to Koshy et al. (2000), children make different types of mistakes when they are engaged in mathematical activities. It is possible for them to make an error where they have a good understanding of an idea but they are perhaps distracted and something goes wrong. However, a mistake might instead be the result of a misconception. These can be described as partial conceptions based on incomplete or immature reasoning (Ryan and Williams, 2010; Swan, 2003; Thompson, 2008a) and ideas about mathematics resulting from incorrect assumptions or overgeneralisations. Swan clarified that few misconceptions are completely wrong and they often apply in more limited circumstances. An example of this is the view that multiplication always makes a number larger. This works for numbers greater than 1, but not for numbers less than or equal to 1, for example $8 \times {}^{1}/_{2} = 4$. Ryan and Williams (2007) describe misconceptions as 'the result of intelligent engagement'. Barmby et al. (2009: 4–5) regard misconceptions as 'evolving understandings' and as important to children's progression in learning mathematics; 'working through misconceptions is an important part of the process of developing understanding'. With children in the 0–8 age phase, Thompson (2008a: 207) suggests that misconceptions can be thought of as 'limited conceptions'. Challenging misconceptions is necessary so that children can progress with their mathematics and later ideas are built on firm foundations.

Drews (2011) highlighted that it is essential to devote time to confronting misconceptions because deep-rooted ideas will be hard to shift and require a change of mind-set. She recommends that children should be rewarded for having the courage to test out mathematical ideas and this is more important than getting correct answers or presenting work neatly. If misconceptions are not tackled, it could be counterproductive to future learning and progress.

Littler and Jirotkova (2008) warn that adults involved in teaching mathematical ideas should not be tempted to teach short cuts that can later lead to problems for children, as un-learning ideas is very difficult. An example of this is that it can be tempting to teach children that to multiply by 10 they can 'add a zero', however this will not work once a child is multiplying numbers with decimals. The assessment of children's misconceptions is explored in Chapter 7. Specific misconceptions and errors are discussed in the chapters of Part 2 of this book.

## Summary

This chapter has explored why it is important that children learn mathematics and described principles, based on theories, of how children learn mathematics. The role of adults in supporting learning and maximising opportunities within children's everyday lives, touching on their play and the routines they participate in, has been discussed. The importance of building on children's interests in order for mathematical learning to be relevant, interesting and engaging and of enabling positive attitudes was emphasised. Progression in learning mathematics is essential as children get older and move through into later primary and secondary education and then for their lives as adults. Children learn through mistakes and misconceptions and adults can use these in positive ways to maximise effective learning.

## Further reading

Thompson, I. (2008) *Teaching and Learning Early Number*, 2nd edn. Maidenhead: Open University Press.

Sections 1, 2 and 7 are relevant to the ideas discussed in this chapter. It is a readable and accessible book and will give you more detail in many areas.

Carruthers, E. and Worthington, M. (2006) *Children's Mathematics: Making Marks, Making Meaning*, 2nd edn. London: Sage.

Mathematical schemas are explored in Chapter 3 of this book. It will help you to put the idea of building on children's interests into action. Other chapters in the book will also be of interest such as bridging the gap between home and school mathematics.

MacGregor, H. (1998) *Tom Thumb's Musical Maths*. London: A and C Black.

In this book new words have been written to familiar tunes to aid mathematical learning.

Moseley, C. (2010) 'Stories for primary mathematics', *Mathematics Teaching*, 219: 16–17.

Moseley suggests some stories to use as starting points and what mathematics may be accessed through them.