

# 6

## Groups and Positions in Complete Networks

### **OBJECTIVES**

The objective of this chapter is to show how a complete network can be analyzed further by using different algorithms to identify its groups and positions. You will also evaluate the difference between the “top-down” and “bottom-up” approaches through which groups are identified. Unlike most social science, the idea is to identify these groups through their relational data, not an exogenous attribute such as grade level, departmental affiliation, or years of experience. After reviewing these different approaches, you will learn how the concept of group differs from position. Positions are also identified through relational data, and you will learn the different ways through which this is done. These different ways are based on contrasting definitions of equivalence, a key concept that you will encounter at the end of the chapter. Finally, once you examine the different ways in which positions are identified, you will then see how relations among different positions can be analyzed and visually represented.

## GROUPS

Once you have a handle on the network's topography using some combination of the structural properties covered in Chapter 5, complete-level network analysis typically moves on to an analysis of the network's substructures. It is at this point in the analysis that you seek to identify those larger groupings that give contours to the network's topography. This section follows the lead of Hanneman and Riddle (2011b) by looking at these groups from either the "top-down" or "bottom-up." More specifically, through the bottom-up approach, you can identify groups that are built up from simple dyads and triads that extend into dense clusters that give the network its "clumpiness." Attention in this approach is given to how larger structures are composed of smaller groups of actors. Similar to complete network measures, there are several important properties for which precise definitions and algorithms have been developed, including cliques, clans, plexes, and cores.

### "TOP-DOWN" APPROACHES TO GROUP ANALYSES

The "top-down" approach focuses on how larger structures are built from smaller ones (Hanneman & Riddle, 2011b). Therefore, this approach takes the complete network and considers parts of it that are dense and somewhat distinct from the rest of the network. These "parts" can be defined and measured in different ways, including components, bi-components, and factions. However, the approach that you use, whether top-down or bottom-up, will likely yield different groups. The important thing to consider when selecting a method is your definition of what constitutes a meaningful group and then to select the algorithm that best matches this definition.

To illustrate these ideas about groups, this section will again rely on the Fraternity Data, a binary, directed set of complete network data. When permitted by the algorithms, these directed data will be used. However, not all algorithms can handle (or are appropriate for) directed data. Therefore, in these instances, symmetrical data will be used; a friendship tie exists between each pair if either  $x_{ij} = 1$  or  $x_{ji} = 1$ . It should be noted that conceptually, this is suspect; to consider that a friendship exists between two students because one nominates the other is a stretch. But, for the purposes of presentation, those algorithms that require symmetrical data will rely on this transformation. These symmetric data are presented in Table 6.1, which is simply an undirected version of the relational data in the sociomatrix in Table 5.2. If a student either sent or received a friendship nomination, the symmetrical sociomatrix represents this as a reciprocated tie. Using these symmetrical data, this section first reviews the three different definitions of substructures identified through the top-down approach and then moves on to the four ways associated with the bottom-up approach.

**Table 6.1** Symmetrized Fraternity Data Sociomatrix. These transformed symmetric data are an undirected version of the relational data in the sociomatrix in Table 5.2. It is sometimes necessary to use undirected data such as these in a group or positional analysis. Transforming these relational data from directed to undirected is typically done through a general social network software package.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1
2	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0
3	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
4	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1
5	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1
6	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0
7	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	0	1
8	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	1
10	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0
11	1	0	1	0	1	0	0	1	1	0	0	1	0	0	1	1	1
12	0	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	1
13	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
14	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0
15	0	0	0	0	1	0	0	0	0	1	1	0	1	0	0	0	0
16	0	1	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0
17	1	0	1	1	1	0	1	0	1	0	1	1	0	0	0	0	0

## Components

One of the most basic ways in which network researchers first characterize a network's substructure is to identify its components. It may be helpful to consider this concept in terms of graphs (Chapter 3). A component is a connected subgraph in which

there is a path between all pairs of nodes (Wasserman & Faust, 1994); that is, all pairs in the component are reachable. If a graph has one component, the graph is considered connected. If it has more than one component, it is disconnected. Each isolated node (an actor with zero ties) is considered its own component. Directed graphs, such as the graph that constitutes the Fraternity Data, have two different kinds of components: weak and strong. A weak component ignores the direction of a tie; strong components do not. Stated differently, strong components consist of nodes that are connected to one another via both directions along the path that connects them. Weak components consist of a set of nodes that are connected regardless of the direction of the ties. This concept of a component is simple and very useful, especially when analyzing a large network with many components. Typically, researchers extract each component and analyze it as a separate network.

Researchers are also interested in components for a few different reasons. For example, networks can be described in terms of their number of weak and strong components. A network with one component (in this case, the graph is equal in size to its one component) is very different than a network with many components. Components can also be used to define and describe groups within a network. If there is a path connecting two nodes, you can describe them as being part of the same group. This first pass can then be used to further identify groups within each component using different and stricter definitions.

Because the Fraternity Data are directed with friendship relations being sent from one student to another, it makes sense to first analyze the network's strong components. In these data, there are four strong components: two are isolates, one has nine members, and the other has six. However, if you are interested in the network's weak components, thereby ignoring the direction of the tie, the network has one component consisting of all 17 students, which is expected given that each student was initially asked to rank the others. If these data were valued—that is, the cells of the sociomatrix had values ranging from  $0 \dots x$ —it would be necessary to define what is known as a cut-off value. A cut-off value is determined by you from valued data to identify whether a relationship between two nodes exists or not. The selection of this value should not, however, be arbitrary. It should be informed by theory (for example, how often does one have to study with another in order to be identified as study partners?) and subject to sensitivity analyses. Continuing to think of the network's substructures in terms of graphs, there are several other ways to define groups within a network.

## Bi-Components

Bi-components are an especially useful way of identifying the important “weak” spots in a graph (Hanneman & Riddle, 2011b). This technique allows you to ask what would happen to the network if an actor were removed. Would the network become

divided into disconnected segments? Nodes that serve this purpose are referred to as “cutpoints” and are usually important actors; that is, they “keep things together.” When a graph is divided in such a manner, it is referred to as bi-component (also called “blocks,” which are different than the blocks identified in a positional analysis discussed later this chapter). Applying this idea to the symmetrized Fraternity Data, there is only one bi-component. This parallels the early finding of the network’s relatively low centralization (CD) score. No one student is responsible for keeping the network connected.

## Factions

Factions are a third means through which a network’s substructures can be defined through a top-down approach. In addition, this procedure also indicates the density within the groups that have been identified. However, before describing how this works, consider the following hypothetical example. There is a classroom (graph) in which each student (node) was closely connected to all the others in his or her group (subgraph) and there are no connections between these groups, with each group therefore being its own component. Of course, most classrooms or real social systems do not look like this, but this hypothetical example is a useful frame of reference for assessing a network’s factions.

If the students in each faction were then placed (permuted) in their own rows and columns in an adjacency matrix, you could then see a pattern of blocks consisting of either 1s or 0s (referred to as 1-blocks or 0-blocks). All connections within a 1-block would be present and connections between factions (blocks) would be absent. Running this routine on the symmetrized Fraternity Data reveals several interesting structural properties that are reported in the UCINET output, Figure 6.1. The number of “blocks” to enter is up to you, and after working upward from two, it was decided that three was a meaningful number of blocks.

The UCINET output provides results regarding the routine’s “goodness of fit,” a permuted adjacency matrix, and block densities. The “final proportion correct” can be thought of as a goodness-of-fit measure. This value, 0.79, reflects the sum of the number of 0s within factions (where all the ties in the ideal type are supposed to be present) and the number of 1s in the nondiagonal blocks (ties between members of different factions, which are supposed to be absent in the ideal type) divided by the networks total number of ties (51). This is a pretty good fit. The three factions are identified, with Students 2, 4, 7, 14, and 16, for example, belonging to faction #3. The grouped adjacency matrix shows the permuted solution. On the diagonal of this matrix, it is evident that most of the relations are present (1s), indicating that relations within these groups are dense. The final panel in this output confirms this point by reporting the block densities: the number of ties in the block as a proportion of all possible ties. For example, within the first block, 80% of relations are present, whereas only 14% are present between the first and second blocks.

**Figure 6.1** Output from UCINET's Factions Routine on the Symmetrized Fraternity Data. Three factions have been identified with Students 2, 4, 7, 14, and 16, for example, belonging to faction #3. The grouped adjacency matrix shows the permuted solution. On the diagonal of this matrix, it is evident that most of the relations are present (1s), indicating that relations within these groups are dense. The final panel in this output confirms this point by reporting the block densities: the number of ties in the block as a proportion of all possible ties.

FACTIONS

-----  
 Number of factions: 3  
 Measure of fit: Hamming  
 Input dataset: NEWCOLE3-Sym (C:\Program Files\Analytic Technologies\Ucinet 6\  
 DataFiles\chap 5\NEWCOLE3-Sym)

Initial proportion correct: 0.684

...Badness of fit: 58.000  
 ...Badness of fit: 58.000  
 ...Badness of fit: 58.000

Final proportion correct: 0.787

Group Assignments:

1: 3 5 9 11 12 17  
 2: 1 6 8 10 13 15  
 3: 2 4 7 14 16

Grouped Adjacency Matrix

1	1 1	1	1 1	1 1
9 2 3 5 1 7	0 6 8 5 3 1	2 4 4 6 7		
-----				
9	1	1 1		1 1
12	1	1 1 1 1		1
3	1	1 1		
5	1	1 1	1	
11	1 1 1 1	1	1 1 1	1
17	1 1 1 1 1		1	1 1
-----				

```

10 |           |      1 1 1 |      1 1 |
 6 |           |      1 1 1 |      1     |
 8 |           |     1 1     |           |
15 |           |     1 1     |      1     |
13 |           |      1 1 1 |           |
 1 |           |     1 1 |     1     |
-----
 2 |           |           |     1 1 1 |
 4 |           |     1     |     1 1 1 |
14 | 1         |     1     |           |
16 | 1         |     1     |     1 1     |
 7 | 1         |     1 |     1     |     1 1 1 |
-----

Density Table

      1     2     3
    ---- ---- ----
 1  0.80 0.14 0.20
 2  0.14 0.47 0.10
 3  0.20 0.10 0.60

```

There are two other relatively recent top-down approaches that identify a network's substructures that deserve a brief mention. While the algorithms behind both approaches are complex, the intuitive logic undermining them is very appealing. The first is the Girvan-Newman (GN) technique (2002; Newman & Girvan, 2004), which emphasizes those ties between actors that, if removed, would partition the network into mutually exclusive groups. In general, GN subgroups are identified by first calculating betweenness centrality (this version of centrality is discussed in Chapter 7) on the ties, and, second, determining if there are any components revealed once ties with the highest betweenness scores are removed. This process is repeated until the number of desired groups is obtained. Similar to the factions routine, you can predetermine the number of groups. Or the analyses can proceed until no groups greater than a certain size are detected. The GN technique, in addition to partitioning the network into some number of mutually exclusive groups, also measures how well the partition characterizes the network. This measure is called modularity (Newman & Girvan, 2004). The equation it provides calculates what is referred to as  $Q$ , the percentage of ties in the network that occur within the subgroups found by the GN algorithm. Therefore, a  $Q$  of 100% reflects that all the ties are to actors within the groups. There is, however, no current standard for an acceptable  $Q$  value. Regardless, the algorithm provides a nice way to divide the network into mutually exclusive subgroups and index how well the partition captures the

network's pattern of ties. Therefore, you have the ability to choose among competing partitions derived from this algorithm and select the one that best fits.

A second noteworthy top-down approach is Moody and White's (2003) technique for identifying nested cohesive subgroups. This approach differs significantly from those already discussed in one important way. Whereas routines such as factions and GN partition the network's actors into mutually exclusive groups (i.e., each actor belongs to one group), the algorithm developed by Moody and White identifies hierarchies of nested cohesive groups. In essence, this approach permits actors to be embedded in more than one group simultaneously, which is a closer approximation of what social life looks like. This algorithm is based on the removal of individual nodes and the identification of  $K$ -components (maximal  $K$ -connected subgraphs, where  $K$  equals the number of nodes specified by you). Both the GN subgroups and Moody and White cohesive subgroups, though computationally complex, represent significant advances in identifying a network's subgroups through a top-down approach.

## “BOTTOM-UP” APPROACHES TO GROUP ANALYSES

### *K*-Cores

The first of four different ways in which a network's substructures can be identified through a more “bottom-up” approach is through  $K$ -cores. Recall that this bottom-up approach starts first with the dyad and extends upward (Hanneman & Riddle, 2011b). The network's overall structure, then, is viewed as emerging from overlaps of the graph's smaller parts.  $K$ -cores reflect this logic. Whereas a component consists of all the nodes that have at least one connection, a  $K$ -core is a substructure (a subgraph) of the network in which each node within the  $K$ -core is connected to at least  $K$  other nodes (Valente, 2010). Therefore, a  $3K$ -core is a substructure, a subset of actors, in which each node is connected to at least three other nodes; a  $2K$ -core would be a subset in which a node is connected to two others, and so forth. Those nodes that do not meet  $K$ , which is defined by you, are dropped from the network. As you increase the value of  $K$ , the remaining relations will appear increasingly dense as less-connected others are removed from the network. A technique that is often used is what is called a  $K$ -core collapse: what happens to the pattern of nodes as  $K$  increases. By charting this process, you are able to identify whether there is a “core” group of actors at the center of the network, while others are on the periphery.

The  $K$ -core routine is especially helpful when dealing with larger networks of actors (unlike the Fraternity Data set). It is also intuitively appealing: If an actor is connected to a sufficient number of other actors in the network, they may feel as though they belong to that group even if they are not directly connected to many or even most members. From this perspective, one's membership in a group is based on connections rather than immersion in a subgroup, as is the case with cliques.



## *N*-Cliques

*K*-cores allow you to identify the network's core group of actors, which may be of empirical interest. However, you might also want to reveal how groups are distributed in the network and which actors belong to which groups. A clique analysis is one way to satisfy these purposes. A clique is a maximally connected subgraph of nodes ( $> 2$ ) in which all nodes are connected to each other.

Figure 6.2 shows the UCINET output from the clique analysis. In the symmetrized Fraternity Data, there are 11 cliques. There are three cliques that consist of four students, with students in multiple cliques. For example, Student 11 is in six cliques (cliques 1–6). The bottom section of the output shows each student's proximity to each of the 11 cliques (or how “adjacent” they are to the clique). Student 1 is adjacent to half (0.50) of the members of clique 1, and Student 3 is adjacent to more than three-quarters of the members in clique 1 (0.75). There is a fairly high degree of co-clique membership in these data. Again, this can be somewhat expected, as these relations were measured at the beginning of the study. You can reasonably expect that as the semester progresses, this degree of overlap would change.

**Figure 6.2** Output from UCINET's Clique Routine on the Symmetrized Fraternity Data. This output shows that there are three cliques that consist of four students, with students in multiple cliques. For example, Student 11 is in six cliques (cliques 1–6). The bottom section of the output shows each student's proximity to each of the 11 cliques (or how “adjacent” the student is to the clique). Student 1 is adjacent to half (0.50) of the members of clique 1, and Student 3 is adjacent to more than three-quarters of the members in clique 1 (0.75).

```

CLIQUES
-----

Minimum Set Size: 3
Input dataset: NEWCOLE3-Sym (C:\Program Files\Analytic Technologies\
Ucinet 6\DataFiles\chap 5\NEWCOLE3-Sym)

11 cliques found.

    1:  9 11 12 17
    2:  5 11 12 17
    3:  3 11 12 17

```

(Continued)

**Figure 6.2** (Continued)

```

4:  1 11 17
5:  5 11 15
6:  9 11 16
7:  2 4 7
8:  2 4 16
9:  4 7 17
10: 7 10 14
11: 7 12 17

```

Clique Participation Scores: Prop. of clique members that each node is adjacent to

	1	2	3	4	5	6	7	8	9	10	11
1	0.500	0.500	0.500	1.000	0.333	0.333	0.000	0.000	0.333	0.333	0.333
2	0.000	0.000	0.000	0.000	0.000	0.333	1.000	1.000	0.667	0.333	0.333
3	0.750	0.750	1.000	0.667	0.333	0.333	0.000	0.000	0.333	0.000	0.667
4	0.250	0.250	0.250	0.333	0.000	0.333	1.000	1.000	1.000	0.333	0.667
5	0.750	1.000	0.750	0.667	1.000	0.333	0.000	0.000	0.333	0.000	0.667
6	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.333	0.333	0.000	0.000
7	0.500	0.500	0.500	0.333	0.000	0.000	1.000	0.667	1.000	1.000	1.000
8	0.250	0.250	0.250	0.333	0.333	0.333	0.000	0.000	0.000	0.333	0.000
9	1.000	0.750	0.750	0.667	0.333	1.000	0.000	0.333	0.333	0.333	0.667
10	0.000	0.000	0.000	0.333	0.333	0.000	0.333	0.000	0.333	1.000	0.333
11	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.333	0.333	0.000	0.667
12	1.000	1.000	1.000	0.667	0.667	0.667	0.333	0.000	0.667	0.333	1.000
13	0.000	0.000	0.000	0.333	0.333	0.000	0.000	0.000	0.000	0.000	0.000
14	0.250	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.333	1.000	0.333
15	0.250	0.500	0.250	0.333	1.000	0.333	0.000	0.000	0.000	0.333	0.000
16	0.500	0.250	0.250	0.333	0.333	1.000	0.667	1.000	0.333	0.000	0.000
17	1.000	1.000	1.000	1.000	0.667	0.667	0.667	0.333	1.000	0.333	1.000

The definition for a clique, however, is very strict. Does each member of the clique need to be connected to all others in order for a group to be defined as such? This definition is likely too strong for most purposes and poorly reflects interaction within social groups. Rarely do members of a group have direct ties with each and every member. To relax this definition, researchers often use what is referred to as an  $n$ -clique, where  $n$  equals the maximum lengths of paths to all other members. Typically, a path distance of two is used, corresponding to the “friend of a friend” idea (Hanneman & Riddle, 2011b).

Therefore, a  $2n$ -clique is a set of actors connected to each other within two steps. This permits actors to be identified as being part of the same group even if they are not directly tied to each other. You can also increase the value of  $n$ , but this is not advisable, as it seems odd for actors to be in the same clique if they are three steps from one another. Figure 6.3 reports the  $2n$ -cliques for the symmetrized Fraternity Data. By relaxing the criterion for group membership, the number of cliques has increased from 11 to 20, with Student 1, for example, being a member of 16 cliques.

**Figure 6.3** Output from UCINET's  $2n$ -Clique Routine on the Symmetrized Fraternity Data. A  $2n$ -clique is a set of actors connected to each other within two steps. This permits actors to be identified as being part of the same group even if they are not directly tied to each other. By relaxing the criteria for group membership, the number of cliques has increased from 11 to 20, with Student 1, for example, being a member of 16 cliques.

```

N-CLIQUEs
-----
Max Distance (n-): 2
Minimum Set Size: 3
Input dataset: NEWCOLE3-Sym (C:\Program Files\Analytic Technologies\
Ucinet 6\DataFiles\chap 5\NEWCOLE3-Sym)

20 2-cliques found.

 1:  1 3 4 5 7 8 9 11 12 16 17
 2:  1 4 5 7 8 9 10 11 12 17
 3:  1 4 7 8 9 10 11 12 14 17
 4:  1 4 7 8 9 11 12 14 16 17
 5:  1 4 6 7 8 10 11 17
 6:  1 4 6 7 8 11 16 17
 7:  2 4 7 9 10 11 12 14 17
 8:  2 4 7 9 11 12 14 16 17
 9:  2 4 6 7 11 16 17
10:  2 4 6 7 10 11 17
11:  1 4 6 8 10 11 13 17
12:  1 4 5 8 10 11 13 17
13:  1 3 5 7 8 9 11 12 15 16 17
14:  1 5 7 8 9 10 11 12 15 17
15:  1 7 8 9 10 11 12 14 15 17

```

(Continued)



easy to find by examining  $n$ -cliques and then eliminating those with a diameter more than  $n$ . However, despite the simplicity of this idea and its availability in social network analysis software packages, this technique is not used often.

## ***K*-Plexes**

The  $K$ -plex is another alternative way of relaxing the rigid criterion for clique membership and is more commonly used than  $n$ -clan method. One reason for its use is that it nicely reflects real-life group structures in that it requires actors associated with a group to be connected to most of that group's members and that a connection through a nonclique intermediary does not qualify an actor for group membership (Hanneman & Riddle, 2011b). A  $K$ -plex, therefore, can be defined as the set of actors connected to all but  $K$  other actors in the group. To find a network's  $K$ -plexes, you set the value of  $K$  and  $n$  to the size of the groups, where the minimum size of  $n$  is set to  $K - 2$ . For example,  $3K$ -plexes with  $n = 10$  will find all of groups with 10 actors in which each actor is connected to at least seven others in the group. If  $K$  is increased to four,  $4K$ -plexes, all groups of size 10 in which each actor is connected to at least six others will be reported. Therefore, as the value of  $K$  increases, so too does the number of groups identified. Valente (2010) notes that in practice, you set  $K$  and finds all the groups as  $n$  increases from  $K + 2$  to  $n - 1$ .

Figure 6.4 reports the  $2K$ -plexes in the symmetrized Fraternity Data. It is evident from this output that the image of group structure that results from a  $K$ -plex approach is vastly different from that of the  $2n$ -clique approach (Figure 6.3): with  $K = 2$  and  $n = 4$ , the algorithm returns 28 different subgroups. It is also evident that these results demonstrate that actors are embedded in "overlapping social circles," which more closely reflects individuals' interactions with social groups. For example, Student 11 is in 17 different  $2K$ -plexes.

These bottom-up approaches— $K$ -cores,  $N$ -cliques,  $N$ -clans, and  $K$ -plexes—provide different ways to understand the subgroups within a network. Similar to the three top-down approaches (components, bi-components, and factions), these different subgroup identification methods allow you to characterize the network according to patterns of who interacts with whom. Though each procedure will provide you with a different take on the network's substructures, they are all based on the ways in which actors are interconnected. These connections within and between groups influence an array of social processes relevant for educational researchers, including solidarity, shared norms, trust, identity, and collective behavior. Identifying these groups through the different techniques introduced here is a means to an end rather than an end in itself. The ultimate goal of using information about a network's substructures is to test whether attitudes and behaviors differ within and between these groups. This brief introduction merely introduced ways to identify and describe these subgroups. Chapter 9 introduces ways to more formally test whether there is a relationship between these groups and outcomes

**Figure 6.4** Output from UCINET's  $2K$ -plexes Routine on the Symmetrized Fraternity Data. The  $K$ -plex is an alternative way of relaxing the rigid criterion for clique membership and is more commonly used than the  $n$ -clan method. One reason for its use is that it nicely reflects real-life group structures in that it requires actors associated with a group to be connected to most other group members, and a connection through a nonclique intermediary does not qualify an actor for group membership. It is evident from this output that the image of group structure that results from a  $K$ -plex approach is vastly different from that of the  $2n$ -clique approach (Figure 6.3): with  $K = 2$  and  $n = 4$ , the algorithm returns 28 different subgroups.

```

K-PLEX
-----

Value of K:                2 (each member of a K-plex of size N has
N-K ties to other members)
Minimum Set Size =         4
Input dataset:             NEWC0LE3-Sym (C:\Program Files\Analytic
Technologies\Ucinet 6\DataFiles\chap 5\NEWC0LE3-Sym)

28 k-plexes found.

1:  1 3 11 17
2:  1 5 11 17
3:  1 7 10 17
4:  1 8 10 11
5:  1 9 11 17
6:  1 10 11 15
7:  1 10 13 15
8:  1 11 12 17
9:  1 11 13 15
10: 2 4 7 16
11: 2 4 7 17
12: 3 5 11 12 17
13: 3 7 12 17
14: 3 9 11 12 17
15: 4 7 12 17
16: 4 9 16 17
17: 4 11 16 17
18: 5 7 12 17
19: 5 9 11 12 17
20: 5 11 12 15

```

```

21: 5 11 15 17
22: 7 9 12 14
23: 7 9 12 17
24: 7 9 14 17
25: 7 11 12 17
26: 8 10 11 15
27: 9 11 12 16
28: 9 11 16 17
    
```

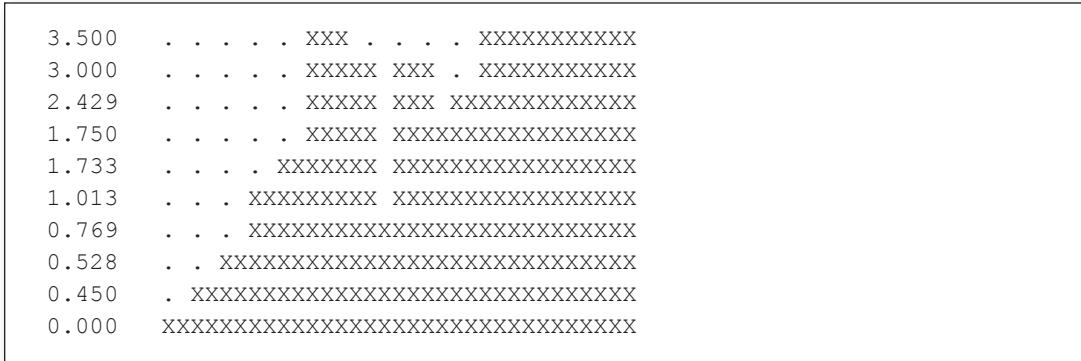
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	9	0	1	0	1	0	1	1	1	4	7	1	2	0	3	0	5
2	0	2	0	2	0	0	2	0	0	0	0	0	0	0	0	1	1
3	1	0	4	0	1	0	1	0	1	0	3	3	0	0	0	0	4
4	0	2	0	5	0	0	3	0	1	0	1	1	0	0	0	3	4
5	1	0	1	0	6	0	1	0	1	0	5	4	0	0	2	0	5
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	2	1	3	1	0	10	0	3	1	1	6	0	2	0	1	8
8	1	0	0	0	0	0	0	2	0	2	2	0	0	0	1	0	0
9	1	0	1	1	1	0	3	0	9	0	5	5	0	2	0	3	7
10	4	0	0	0	0	0	1	2	0	5	3	0	1	0	3	0	1
11	7	0	3	1	5	0	1	2	5	3	17	7	1	0	5	3	11
12	1	0	3	1	4	0	6	0	5	0	7	12	0	1	1	1	9
13	2	0	0	0	0	0	0	0	0	1	1	0	2	0	2	0	0
14	0	0	0	0	0	0	2	0	2	0	0	1	0	2	0	0	1
15	3	0	0	0	2	0	0	1	0	3	5	1	2	0	6	0	1
16	0	1	0	3	0	0	1	0	3	0	3	1	0	0	0	5	3
17	5	1	4	4	5	0	8	0	7	1	11	9	0	1	1	3	19

HIERARCHICAL CLUSTERING OF OVERLAP MATRIX

Level	6	2	4	8	3	1	0	5	4	6	3	5	7	9	2	1	7
11.000	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	XXX
8.333	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	XXXXXX
6.000	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	XXXXXXXX
5.200	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	XXXXXXXXXX
4.000	.	.	.	.	.	XXX	.	.	.	.	.	.	.	.	.	.	XXXXXXXXXX

(Continued)

**Figure 6.4** (Continued)



relevant to educational researchers. For example, is a cluster of highly effective teachers within a school more likely to socialize with another cluster of highly effective teachers?

To answer such a question, you must first identify a network’s subgroups through one of the techniques introduced above. In exploratory research, it is recommended that you first look for components, then apply *K*-cores, and then search for complete triads to further subdivide large *K*-cores, if necessary. The decision tree presented in Figure 6.5 is designed to help you make some of these decisions as you seek to identify the groups within a network. These decisions should also be guided by a form conceptualization of what constitutes a “group.” This requires that the analytic decision be theoretically justifiable.

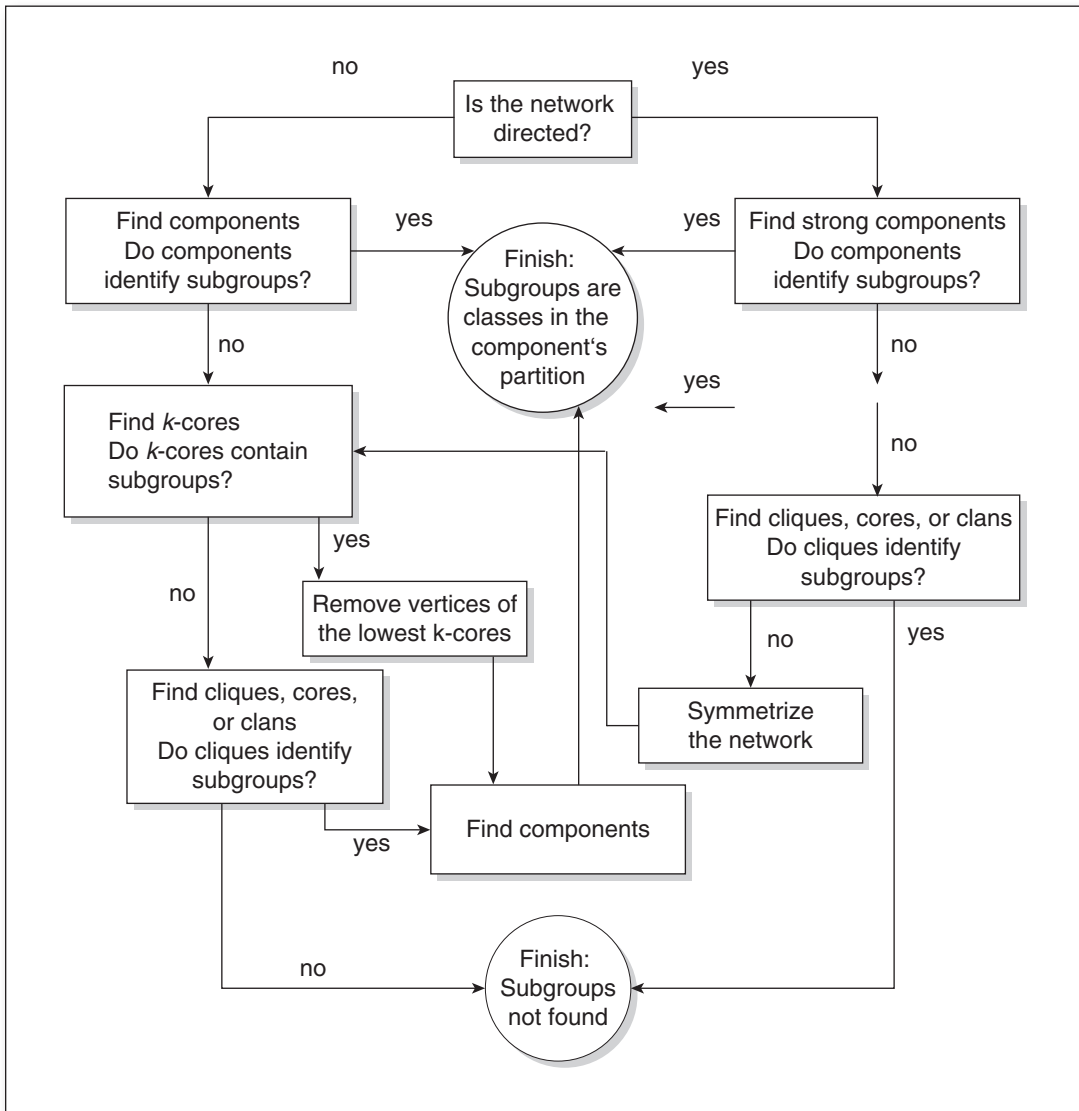
## POSITIONS

In addition to examining a complete network’s structural properties and the groups within that network, analysts are often interested in identifying the positions in a network and the relationships between these positions. Similar to describing a network’s structural properties, a complete network-level positional analysis is performed using mathematical algorithms to find positions and their relationships. Positions, however, differ from groups. Groups, as described above, are collections of actors who share some relation or are connected (degree) to each other at a higher rate (density). A network position, on the other hand, is a set of actors that occupy the same place or have similar patterns of relations with others. So positions consist of actors who share the same place in the network, but they need not be connected to each other, though they very well might be.

Broadly speaking, a position is a set of actors that have the same connections to similar others or types of others. For example, two veteran teachers in the same school



**Figure 6.5** Decision Tree for the Analysis of a Network's Groups. In exploratory research, it is recommended that one first look for components, then apply *K*-cores, and then search for complete triads to further subdivide large *K*-cores, if necessary. Working from the top down, follow the steps in this decision tree to help you explore a complete network's group structures.



Source: de Nooy, W., Mrvar, A., & Batagelj, V. (2005). *Exploratory social network analysis with Pajek*. New York: Cambridge University Press.

(the school being the network's boundary) share the same position (veteran teacher), but they need not share a relation of any kind. The fact is that they occupy this position because they both report to assistant principals and mentor junior teachers. These two teachers are alike even though they mentor and report to different others. Given the school as a social system, a veteran teacher is a position and the veteran teacher's roles include mentoring younger colleagues, among others. The assistant principal is another position that carries expectations of appropriate leadership conduct toward both veteran and junior teachers. Expectations for teachers and assistant principals—as well as all the other positions in the network—are coupled into a system of roles.

Determining these positions, as well as the relations between them, consists of three general steps (Valente, 2010): (1) using mathematical algorithms to identify unique positions; (2) examining how these positions relate to each other; and (3) determining how these positions influence behaviors and attitudes. This section introduces the idea of blockmodeling, the primary means through which network researchers perform a positional analysis; the difference across these blockmodeling techniques is the way in which the various algorithms identify the positions. Positional analysis was a popular strand of network research up to early 1990s; however, there are few, if any, recent examples of this type of analysis in educational research. Therefore, this discussion on positional analysis will be kept brief.

## Equivalence

A positional analysis is grounded in the concept of equivalence. Equivalence, in general, refers to actors who occupy the same position. But, similar to the varied ways in which groups can be defined, there are different ways in which the concept of equivalence is defined. These definitions, in turn, lead to the identification of different positions. The most restrictive of these definitions is structural equivalence: In a directed, binary graph, two actors are structurally equivalent on a specific relation if they have exactly identical patterns of ties sent to and received from all the other actors in the network.

Real network data, however, rarely consist of dyads that meet this rigorous standard for equivalence (identical ties to and from identical others in the same graph). A less restrictive definition, which also applies only to a single graph (network), is automorphic equivalence. Two actors are automorphically equivalent if they are connected to other corresponding positions but not to identical actors. For example, for two high school teachers to occupy a structurally equivalent position, both teachers must teach the same set of students, something that is highly unlikely. But, to hold an automorphically equivalent position, the two teachers need only teach different sets with the same number of students. The students, therefore, occupy a second position that is defined by what they do—receive instruction from teachers. This brings to mind an important point about

positional analysis. Positions are established through relational data, not exogenous attributes. Related to this definition of equivalence and often incorrectly interchanged (Borgatti & Everett, 1992a) is isomorphic equivalence, which applies to two networks of actors. In practice, this means that if two actors are connected in one network, then the corresponding actors in the other network must also be connected in the same way.

The least restrictive of these definitions is regular equivalence, which requires neither structural equivalence's ties to identical actors nor automorphic equivalence's indistinguishable positions (Knoke & Yang, 2008). Actors are regularly equivalent if they have identical relations to and from equivalent actors (Wasserman & Faust, 1994). For example, two popular elementary school teachers occupy the same social position, though in different grades, because they are well liked by some children and respected by some parents, but they do not teach the same kids nor are they respected by the same parents. Again, the important point is that their position of "popular teacher" is established by relational data (affinity and respect), not characteristics that are removed from context.

To demonstrate the difference among these three equivalence definitions, Figure 6.6 shows a hypothetical hierarchy of a school district's organizational chart, which consists of three levels linked by supervisory relation. In this figure, there are seven structurally equivalent positions. Recall that two nodes are considered to be structurally equivalent if they have the same relationships to all other nodes. Therefore, because there is no actor who has exactly the same set of ties as actor A, that one actor is in a class by itself. The same is true for actors B, C, and D. Each of these actors has a unique set of ties to others, so they form three classes, each with one member. Actors E and F, however, fall in the same structural equivalence class. Each has a single tie, and that tie is to actor B. Since E and F have exactly the same pattern of ties with all other actors, they are structurally equivalent. Actor G, again, is in a class by itself; its profile of ties with the other nodes in the diagram is unique. Finally, actors H and I fall in the same structural equivalence class—that is, they have exactly the same pattern of ties to all other actors.

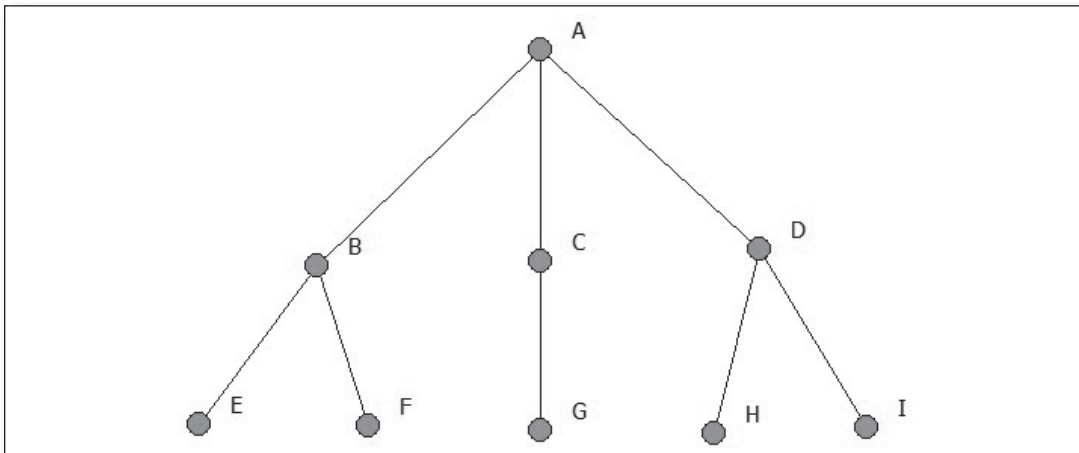
However, if the definition of equivalence were to be relaxed, the positions identified in the graph would be different. Consider that even though actors B and D are not structurally equivalent (they do report to the same supervisor but do not supervise the same actors), they do seem to be "equivalent" in a different sense. Actors B and D can switch locations, but the distances among all the network's actors would remain the same. Therefore, actors B and D form an automorphic class. In Figure 6.5, there are actually five automorphic equivalence positions: {A}, {B, D}, {C}, {E, F, H, I}, and {G}. These positions are groupings whose members would remain at the same distance from all other actors if they were switched and members of other classes were also switched.

The definition of equivalence can be further relaxed by identifying those actors that have the same profile of ties with members of other sets of actors. Two principals, for example, are equivalent because each has a certain pattern of ties with a superintendent and teachers. The principals do not necessarily have ties to the same teachers, however.

That is, they are not structurally equivalent. Because two different principals may supervise different numbers of teachers, they will also not be considered automorphically equivalent. But they are similar because they have the same relationships with some member or members of another set of actors. This is an intuitive and attractive idea. Regular equivalence sets describe the “social roles” that are the basic building blocks of all social institutions. Actors that are regularly equivalent do not necessarily fall in the same network positions or locations with respect to other individual actors; rather, they have the same kinds of relationships with some members of other sets of actors. In Figure 6.6, there are three regular equivalence classes. The first is actor A (superintendent); the second is composed of the three actors B, C, and D (principals); the third is composed of the remaining five actors E, F, G, H, and I (teachers). The easiest class to observe consists of the five actors across the bottom of Figure 6.6 (E, F, G, H, and I). These actors are considered regularly equivalent to one another because they have no tie with any actor in the first class, and each has a tie with an actor in the second class (either B or C or D). Each of the five actors, then, has an identical pattern of ties with actors in the other classes.

Taken together, Figure 6.6 shows the restrictiveness of structural equivalence compared to automorphic and regular equivalence, with the latter being the least restrictive and most intuitively appealing. Regular equivalence is, therefore, the most flexible

**Figure 6.6** A Visual Comparison of Structural, Automorphic, and Regular Equivalence. Consider this a graph of a hypothetical hierarchy of a school district’s organizational chart, which consists of three levels linked by supervisory relation. Depending on your preferred definition of equivalence, different positions will be identified.



Source: Knoke, D., & Yang, S. (2008). *Social network analysis* (2nd ed.). Thousand Oaks, CA: Sage Publications.

method for identifying positions and social roles in a network. The following section briefly describes the general technique for identifying these equivalent classes of actors.

## Blockmodeling

Blockmodeling is the general technique in which the relations among a network's actors can be reduced to a set of positions (blocks) and the relations among these positions also treated as a network (Valente, 2010)—what is often called a reduced-form network. Lorrain and White (1971) were the first to propose that a network could be reduced to a set of blocks and then the relations among these blocks studied; hence the term *blockmodeling*. There are many ways to identify blocks and determine the relations among them (reviewed in Doreian, Batagelj, & Ferligoj, 2005), each varying by how they define equivalent positions. While positional analysis and blockmodeling can quickly get messy and muddled, a short demonstration using the Fraternity Data will illustrate how positions are defined and how such analyses can be useful in educational research.

Recall that the Fraternity Data in Table 5.2 represent a binary and directed friendship network consisting of 17 male students at the start of a semester-long study. Several structural patterns are evident in these data, and it is instructive to reiterate a few of them prior to moving on to the positional analysis. The network has one weak component, and its average path length is slightly more than two, meaning that it takes, on average, about two steps for one student to “reach” any other student. The network is not highly centralized, and fewer than one-third of the ties are reciprocated. As for its substructures, there are 20  $2n$ -cliques, suggesting that there is connectivity across different cliques, with several students belonging to more than one  $2n$ -clique. From this information, you can conclude that (1) actors are not tightly clustered into distinct social groups; (2) influence and/or power are not highly concentrated; and (3) strong, reciprocated relations have yet to emerge. These conclusions provide some clues about what you might expect from a positional analysis of these same data.

## CONCOR Example

Are there any distinct positions in these data that reflect these structural patterns? To address this question, a positional algorithm, CONCOR (“convergence of iterated correlations”), was applied to the data. This algorithm is based on structural equivalence and does a good job of identifying positions based on node similarities. While the technical details about this algorithm are available elsewhere (see, e.g., Breiger, Boorman, & Arabie, 1975), suffice it to say that its appeal is rooted in the fact that it provides an unbiased, mathematical partition of the network into positions that requires little

theoretical or substantive input from the researcher (Valente, 2010). For a more detailed discussion on blockmodeling, as well as its different applications, see Ferligoj, Doreian, and Batagelj (2011).

Figure 6.7 presents the UCINET output for the blockmodel analysis of the Fraternity Data using the CONCOR algorithm. CONCOR allows you to choose one of the partitions that best conforms to your understanding of the data's underlying structure; hence the need to first investigate the network's overall structure and its substructures (groups). It was decided that the third and final partition best reflected the network's positions. The top panel of the output in Figure 6.6 shows a partition diagram, with Level 3 representing this third partition. From this, you can see that Students 12, 11, and 17 are in one position, and 2, 14, and 16 are in another. There is a total of eight positions, with Students 1 and 6 being sole members of two other positions.

The second panel of the output in Figure 6.7 shows the blocked matrix, which shows ties within and between positions; the second step in a positional analysis. For example, Position 3 (Students 5, 3, and 9) sends nine ties to Position 4 (Students 12, 11, and 17). However, Position 3 only receives three ties in return, indicating that there is an asymmetrical relationship between these two positions. You can easily see from this output if there are ties (1s) within and between positions and blanks representing an absence of ties.

To take this further, the bottom panel in Figure 6.7 reports the density within and between blocks. The general rule is to compare densities within and between blocks

**Figure 6.7** Results of the Blockmodel Analysis of the Fraternity Data Using the CONCOR Algorithm. CONCOR allows the researcher to choose one of the partitions that best conforms to the researcher's understanding of the data's underlying structure; hence the need to first investigate the network's overall structure and its substructures (groups). It was decided that the third and final partition best reflected the network's positions. The top panel of the output shows a partition diagram, with Level 3 representing this third partition. From this, one can see that Students 12, 11, and 17 are in one position, and 2, 14, and 16 are in another. There is a total of eight positions, with Students 1 and 6 being sole members of two other positions.

```
CONCOR
```

```
-----
```

```
Diagonal: Ignore
```

```
Max partitions: 3
```

```
Input dataset: NEWCOLE3 (C:\Program Files\Analytic Technologies\Ucinet 6\
DataFiles\chap 5\NEWCOLE3)
```

PARTITION DIAGRAM

```

                                1 1 1   1 1   1 1 1
Level  1 4 7 5 3 9 2 1 7 2 4 6 6 8 5 0 3
-----
      3  . XXX XXXXX XXXXX XXXXX . XXX XXX
      2  XXXXX XXXXXXXXXXXX XXXXXXXX XXXXXXXX
      1  XXXXXXXXXXXXXXXXXXXX XXXXXXXXXXXXXXXXXXXX
    
```

Relation NEWCO  
Blocked Matrix

```

                                1 1 1   1 1   1 1 1
                                2 1 7   2 4 6   6 8 5 0 3
-----
1 | | | | | 1 1 | | | | 1 |
-----
4 | | 1 | | | 1 | 1 | | | | |
7 | | 1 | | | 1 1 | | | | |
-----
5 | | | | | 1 1 1 | | | | |
3 | | | | | 1 1 1 | | | | |
9 | | | | | 1 1 1 | | | | |
-----
12 | | | | 1 | | 1 1 | | | | |
11 | | | | 1 | 1 1 1 | | | | |
17 | | 1 | | 1 | 1 | | | | |
-----
2 | | 1 1 | | | | 1 | | | | |
14 | | 1 | | 1 | | | | | | 1 |
16 | | 1 | | 1 | 1 | | | | |
-----
6 | | 1 | | | | | | 1 | | 1 |
-----
8 | | | | | 1 | | | 1 | | 1 |
15 | | | | 1 | | 1 | | | | 1 |
-----
    
```

(Continued)

Figure 6.7 (Continued)

```

10 | 1 | 1 |      |      |      |      | 1 |      |
13 | 1 |   |      |      |      |      | 1 | 1 |      |
-----

Density Matrix

      1      2      3      4      5      6      7      8
-----
1      0.000 0.000 0.667 0.000 0.000 0.000 0.000 0.500
2 0.000 1.000 0.000 0.500 0.167 0.000 0.000 0.000
3 0.000 0.000 0.000 1.000 0.000 0.000 0.000 0.000
4 0.000 0.167 0.333 0.833 0.000 0.000 0.000 0.000
5 0.000 0.667 0.222 0.111 0.167 0.000 0.000 0.167
6 0.000 0.500 0.000 0.000 0.000      0.500 0.500
7 0.000 0.000 0.167 0.333 0.000 0.500 0.000 0.500
8 1.000 0.250 0.000 0.000 0.000 0.500 0.500 0.000

R-squared = 0.517

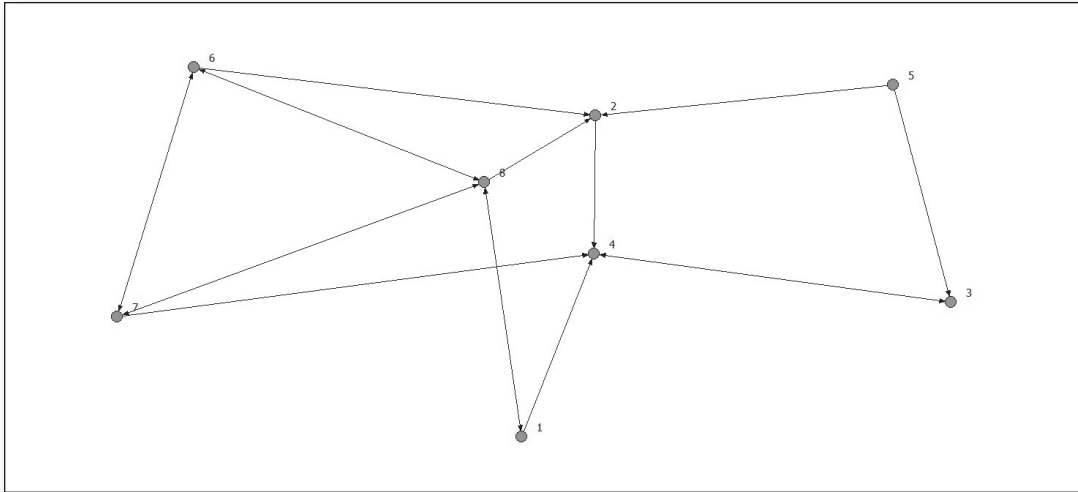
```

with the density of the whole network (overall density = 0.19, reported earlier). If block densities are larger than or equal to this value, you can conclude that there is a connection between these blocks. When densities are less than this value, the blocks (positions) are not connected. In this example, Position 2 is connected to 4 and 5, because the densities are greater than 0.19. Along these same lines, Position 3 is connected only to 4, and so forth. It should be obvious from this that relations between positions do not need to be reciprocated. For example, Position 4 does not send a tie to Position 3.

Now that all 17 students have been assigned to positions and relations between positions have been established, the more substantive work involved with a positional analysis is to label the positions and interpret the relations among them. Rather than refer to these as positions, they can then be characterized by their attributes and/or their relationships with other positions. Because no attribute data on these students are available (e.g., do “smart” students occupy one position?), these positions will be labeled according to how they perform. Figure 6.8 shows the relations among these eight positions. The nodes in this figure are no longer individual students, but rather the positions into which students have been partitioned through the CONCOR



**Figure 6.8** Reduced-Form Network of the Fraternity Data. Positions Identified Through CONCOR. The nodes in this figure are no longer individual students but, rather, the positions into which students have been partitioned through the CONCOR algorithm. From this reduced-form network, it is clear that three positions are vying for control (Positions 2, 4, and 8), with each of those “controlling” access to the periphery positions.



algorithm. From this reduced-form network, it is clear that three positions are vying for control (Positions 2, 4, and 8), with each of those “controlling” access to the periphery positions. These positions and the relations among them conform to other characteristics of the network’s structure. For example, Students 11, 12, and 17 (Position 4) were also those students with the most in-degrees (ties received). Student 17 was also a member of all 20 of the network’s  $2n$ -cliques. When relations between positions are asymmetrical, this usually indicates a relationship based on control or influence, something that Position 4 seems to possess. Position 3 (Students 5, 3, and 9) has a reciprocated relationship with this position, suggesting that this actor, too, may reap some advantages by having a mutual tie with these actors; no other position receives a tie from them.

While positional analyses such as the simple one described above are not often used in educational research, the concepts of positions and roles are central to much social science research (Ferligoj, Doreian, & Batagelj, 2011). Given the advances made in this area, especially in regard to conceptualization, computation, and visualization, it is likely that positional analysis will become a more prominent area for educational researchers. After all, it is one’s position in a social structure that determines expectations for performance, and a great deal of educational research is focused on this (for both

teacher and students). Therefore, performance is not something that is solely attributable to one's individual characteristics. Rather, you need to consider how expectations both shape and are shaped by the social structure of which they are part. Positional analyses are a step in this promising direction.

## SUMMARY

Once a network's topography has been mapped (Chapter 5), the analysis of a complete network generally proceeds to uncover the substructures within that network. These groups are critical, as they provide individual actors with access to resources such as information, advice, and friendship that influence one's own behavior and attitudes. While there are many different ways to define groups using relational data, most agree that a group consists of at least three members who are connected in a way that is greater than their connections to others in the rest of the network.

After the network's substructures are mapped using one of the "bottom-up" or "top-down" approaches discussed in this chapter, the analysis can then proceed to a positional analysis. This chapter emphasized the important conceptual difference between groups and positions, highlighting the different ways in which equivalent positions are defined and ultimately measured. A positional analysis, while not often used in educational research, typically follows the same procedure as a group analysis. First, the positions are identified (the example in this chapter used the CONCOR algorithm based on structural equivalence). Next, the relationships among these positions are mapped from a reduced-form matrix, in which the nodes are positions. Finally, a positional analysis involves linking these positions to outcomes, a step that was not presented in this chapter but could easily be incorporated if you have relevant data available.

## CHAPTER FOLLOW-UP

Once again, select a peer-reviewed empirical research article that employs social network analysis and is related to a topic of your interest. More specifically, locate an article that identifies a network's groups and positions. Use that article to respond to the following questions.

1. How has a group been defined and measured? Were any alternatives considered? Does this definition of a group make sense given the network's actors and relations among them? Is this definition restrictive or flexible?
2. Was a "top-down" or "bottom-up" approach used? Both?

3. How were the network's positions defined and measured? How do these positions differ from the network's groups? Is this difference made evident? What definition of equivalence informed the definition and measurement of the network's positions? Was a reduced-form network presented? What does it indicate about the network's hierarchy?

### ESSENTIAL READING

- Borgatti, S. P., & Everett, M. G. (1992). Notions of position in social network analysis. *Sociological Methodology, 22*, 1–35.
- Lorrain, F. P., & White, H. C. (1971). Structural equivalence of individuals in social networks. *Journal of Mathematical Sociology, 1*, 49–80.
- Moody, J., & White, D. R. (2003). Structural cohesion and embeddedness: A hierarchical concept of social groups. *American Sociological Review, 68*, 103–127.