Introduction: A User's Guide to the Second Edition

Many of you are holding this book because you've become an avid user of the first edition of *Math Tools* (Silver, Brunsting, & Walsh, 2008). This new edition of *Math Tools* has been updated and enhanced due in large part to the emergence of the Common Core State Standards for Mathematics (NGA Center, CCSSO, 2010b). Of course, these new standards (in particular the Standards for Mathematical Practice) do not represent the only impetus for change—if a tool doesn't work well for teachers and students, it has no place in the classroom. So we started developing this second edition of *Math Tools* by reviewing each of the tools in the first. We then sat down with teachers of mathematics who use the tools every day in their classrooms. We retained those tools that work best, revised those tools that could work better, and replaced those that fell a bit short of expectations. We also listened to feedback from the field and tapped in to our own experiences to develop a number of new tools. Below is an overview of the key changes you will notice in this edition.

NEW TO THIS EDITION

- **13 new math tools**, including "Concept Attainment," "Precisely Speaking," and "Thinking Notes" in Chapter 2; "Meaning-Full Math," "Metaphorical Duels," "Modeling With Algebra," and "Modeling With Manipulatives," in Chapter 3; "Explaining Solutions," "I Know What I Know," and "No Uncertain Terms" in Chapter 4; and "Comprehension Menu," "Math Media Skills," and "Math Notes" in Chapter 5

- **Building Common Core Thinking** sections in each chapter that explain how the math tools tie in to and support the relevant Common Core Standards for Mathematical Practice

- **Formative Assessment Connection** and **Technology Connection sidebars** that prompt teachers to link the tools with meaningful assessment techniques and technology exercises

- **Updated matrices** that identify relevant Episodes of Effective Instruction, Sidebars, and the Common Core Standards for Mathematical Practice

- **A reconceived final chapter** devoted to **"mosaic" or multi-style tools** that bring together different styles of thinking from throughout the book
- **A user-friendly companion website** where you can find blank organizers, templates, and other resources to supplement the book content (just look for pages flagged with notes and marginal icons)

While change can be refreshing, we did not want to risk diluting the efficiency or effect of the first edition of *Math Tools.* Thus, we worked hard to ensure that, despite any changes in format, the tools in this second edition of *Math Tools* remain practical, easy-to-use instructional techniques that teachers of mathematics can rely on to differentiate instruction, engage all students, design better lessons and units, and build their students' mathematical reasoning.

WHERE DO THE TOOLS COME FROM?

Building on decades of experience working with teachers and helping schools raise student achievement, Dr. Harvey Silver and the late Richard Strong developed The Thoughtful Classroom—a new approach to professional development committed to "making students as important as standards." One of the hallmarks of The Thoughtful Classroom program has been to develop teachers' repertoires of instructional tools and strategies proven to help students. Over the years, one particular text from The Thoughtful Classroom library has emerged continuously as a go-to resource for this purpose. This award-winning book, *Tools for Promoting Active, In-Depth Learning* (Silver, Strong, & Commander, 1998; Silver, Strong, & Perini, 2001), has always seemed to make the biggest difference in classrooms in the shortest amount of time. The idea behind *Tools for Promoting Active, In-Depth Learning* is wonderfully simple: It is a collection of classroom-tested tools, or straightforward teaching "moves" that teachers can use to foster active, in-depth learning. These tools are based on the principles of effective teaching and learning and require little or no planning. As such, tools can serve as "on-the-fly" techniques whenever a learning episode begins to lag, or they can be planned into a lesson or unit ahead of time to meet specific objectives.

Although *Tools for Promoting Active, In-Depth Learning* was the inspiration for our first edition of *Math Tools,* it is not the only tools book to have informed our thinking for this second edition. A new entry into The Thoughtful Classroom "tools family," *Tools for Thoughtful Assessment* (Boutz, Silver, Jackson, & Perini, 2012), is a collection of tools focused around a dozen important assessment questions teachers address before, during, and after instruction. *Tools for Thoughtful Assessment* reminds us that virtually all tools can be used to assess and develop student learning, so we've borrowed from this new tools format and have committed to highlighting the utility of math tools as assessment tools in this revised edition.

HOW IS THE NEW EDITION OF *MATH TOOLS* ORGANIZED, AND WHAT ARE LEARNING STYLES?

Like the first edition, this second edition of *Math Tools* is organized around different styles of thinking and learning, because teaching the diversity of learners has always been an important goal of ours. And style gives us a powerful and manageable framework for achieving the goal of differentiating instruction.

The history of style stretches all the way back to the work of Carl Jung (1923), one of the founding fathers of modern psychology. What Jung discovered is that the ways in which people acquire and evaluate information tend to affect the development of their preferred ways of thinking and learning. Almost 40 years after Jung published his major work on psychological types, Isabel Briggs Myers and Katharine Cook Briggs (Myers, 1962; Myers, McCaulley, Quenk, & Hammer, 1998) expanded on Jung's work to create a comprehensive model for understanding cognitive diversity. Their work culminated in the world-renowned Myers-Briggs Type Indicator (MBTI), which millions of people take each year to better understand their strengths and liabilities in order to grow as learners, workers, and individuals. In the years since the development of the MBTI, educational researchers—including Bernice McCarthy (1987), Carolyn Mamchur (1996), Edward Pajak (2003), Harvey Silver and Richard Strong (2004), Gayle Gregory (2005), Jane Kise (2007), and Diane Payne and Sondra VanSant (2009)—have explored the implications of these ideas and helped educators apply them to their work in classrooms and schools.

In the interest of making the concept of style more meaningful for mathematics educators, Harvey Silver, Ed Thomas, and Matthew Perini (2008) applied the research on learning styles specifically to the study of mathematics. From this work, they identified four distinct mathematics learning styles (outlined in Figure C).

Figure C The Four Styles of Mathematics Students

Mastery math students . . .	**Interpersonal math students . . .**
• Want to learn practical information and procedures. • Like math problems that are like problems they have solved before and that use set procedures to produce a single solution. • Approach problem solving in a step-by-step manner. • May experience difficulty when mathematics becomes too abstract or when faced with open-ended problems. • Learn best when instruction is focused on modeling new skills, practicing, and feedback and coaching sessions.	• Want to learn mathematics through dialogue, collaboration, and cooperative learning. • Like math problems that focus on real-world applications and on how math helps people. • Approach problem solving as an open discussion among a community of problem solvers. • May experience difficulty when instruction focuses on independent seat work or when what they are learning seems to lack real-world application. • Learn best when their teacher pays attention to their successes and struggles in mathematics.
Understanding math students . . .	**Self-Expressive math students . . .**
• Want to understand why the math they learn works. • Like math problems that ask them to explain, prove, or take a position. • Approach problem solving by looking for patterns and identifying hidden questions. • May experience difficulty when there is a focus on the social environment of the classroom (e.g., on collaboration and cooperative problem solving). • Learn best when they are challenged to think and explain their thinking.	• Want to use their imagination to explore mathematical ideas. • Like math problems that are nonroutine and project-like in nature, and that allow them to think "outside the box." • Approach problem solving by visualizing the problem, generating possible solutions, and exploring among the alternatives. • May experience difficulty when mathematics instruction is focused on drill and practice and rote problem solving. • Learn best when they are invited to use their imagination and engage in creative problem solving.

Source: Adapted from *Math Learning Style Inventory for Secondary Students* (p. 8) by H. F. Silver, E. J. Thomas, and M. J. Perini, 2008, Ho-Ho-Kus, NJ: Thoughtful Education Press. © 2008 Thoughtful Education Press. Used with permission.

These four mathematical styles—Mastery, Understanding, Self-Expressive, and Interpersonal—are each given a chapter of tools and serve as the organizational structure of the second edition of *Math Tools.* The fifth chapter (Mosaic) shows how these different styles can be brought together in individual tools to further enhance instruction and assessment in the mathematics classroom.

- **Mastery** tools support students' acquisition and retention of facts, vocabulary, and step-by-step procedural knowledge crucial to mathematical proficiency.
- **Understanding** tools facilitate the development of students' mathematical reasoning, analysis, and justification skills, which are needed to communicate understanding in a variety of ways.
- **Self-Expressive** tools encourage students to use their imagination to investigate mathematical ideas and creatively solve nonroutine problems.
- **Interpersonal** tools promote authentic student experiences with mathematics by helping them make personal connections between the abstract and the real world.
- **Mosaic** tools are palettes (or arrays)–drawn from aspects of the other four types of tools–that promote differentiated mathematical thinking, asking students to write, make notes, or complete tasks in various ways.

WHAT'S IN A TOOL?

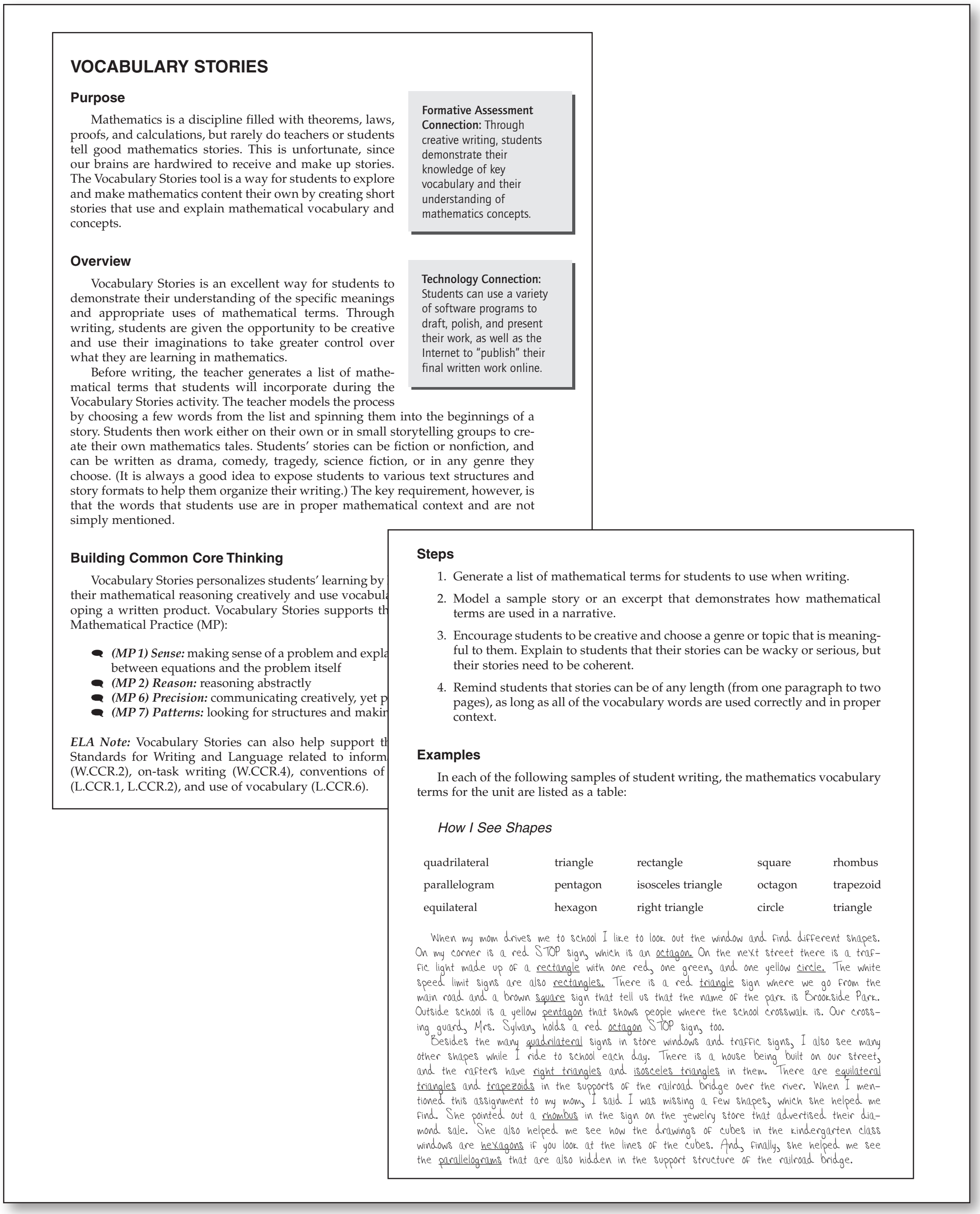
Next, let's look more closely at what's inside each tool. Although there are dozens of tools organized into different styles, each math tool in this book is composed of the same structural elements (see Figure D):

- The **Purpose** highlights why the tool is important, how it is helpful, or what classroom challenge it addresses.
- In the **Overview**, we present a broad but basic picture of how the tool works in the classroom.
- The **Building Common Core Thinking** section illuminates those Standards for Mathematical Practice that the tool supports.
- Straightforward and numbered **Steps** make it easy to implement the tool in any classroom.
- Finally, we provide **Examples**—these can range from simple suggestions or starters to full-length samples of student work.

We wanted not only to ensure that the format of each tool is consistent throughout this new edition of *Math Tools,* but we also wanted to highlight those tools that are particularly well-suited for formative assessment and for connecting to technology or multimedia. So, we developed two new recurring sections—sidebars, really—that we engage for those tools that best highlight **Formative Assessment Connections** and **Technology Connections**.

Of course, the math tools that receive a sidebar are certainly not the only tools that lend themselves to formative assessment or the integration of technology or multimedia into the classroom. We have found that most—if not all—of the tools in the second edition of *Math Tools* can be used for these purposes. We encourage you to treat these sidebars not only as tool-specific recommendations but also as sparks of inspiration that help stimulate your own ideas for how to use tools to formatively assess learning or link technology to instruction.

Figure D A Quick Look at What's in a Tool



VOCABULARY STORIES

Purpose

Mathematics is a discipline filled with theorems, laws, proofs, and calculations, but rarely do teachers or students tell good mathematics stories. This is unfortunate, since our brains are hardwired to receive and make up stories. The Vocabulary Stories tool is a way for students to explore and make mathematics content their own by creating short stories that use and explain mathematical vocabulary and concepts.

Formative Assessment Connection: Through creative writing, students demonstrate their knowledge of key vocabulary and their understanding of mathematics concepts.

Overview

Vocabulary Stories is an excellent way for students to demonstrate their understanding of the specific meanings and appropriate uses of mathematical terms. Through writing, students are given the opportunity to be creative and use their imaginations to take greater control over what they are learning in mathematics.

Technology Connection: Students can use a variety of software programs to draft, polish, and present their work, as well as the Internet to "publish" their final written work online.

Before writing, the teacher generates a list of mathematical terms that students will incorporate during the Vocabulary Stories activity. The teacher models the process by choosing a few words from the list and spinning them into the beginnings of a story. Students then work either on their own or in small storytelling groups to create their own mathematics tales. Students' stories can be fiction or nonfiction, and can be written as drama, comedy, tragedy, science fiction, or in any genre they choose. (It is always a good idea to expose students to various text structures and story formats to help them organize their writing.) The key requirement, however, is that the words that students use are in proper mathematical context and are not simply mentioned.

Building Common Core Thinking

Vocabulary Stories personalizes students' learning by their mathematical reasoning creatively and use vocabul oping a written product. Vocabulary Stories supports th Mathematical Practice (MP):

- *(MP 1) Sense:* making sense of a problem and expla between equations and the problem itself
- *(MP 2) Reason:* reasoning abstractly
- *(MP 6) Precision:* communicating creatively, yet p
- *(MP 7) Patterns:* looking for structures and makir

ELA Note: Vocabulary Stories can also help support th Standards for Writing and Language related to inform (W.CCR.2), on-task writing (W.CCR.4), conventions of (L.CCR.1, L.CCR.2), and use of vocabulary (L.CCR.6).

Steps

1. Generate a list of mathematical terms for students to use when writing.
2. Model a sample story or an excerpt that demonstrates how mathematical terms are used in a narrative.
3. Encourage students to be creative and choose a genre or topic that is meaningful to them. Explain to students that their stories can be wacky or serious, but their stories need to be coherent.
4. Remind students that stories can be of any length (from one paragraph to two pages), as long as all of the vocabulary words are used correctly and in proper context.

Examples

In each of the following samples of student writing, the mathematics vocabulary terms for the unit are listed as a table:

How I See Shapes

quadrilateral	triangle	rectangle	square	rhombus
parallelogram	pentagon	isosceles triangle	octagon	trapezoid
equilateral	hexagon	right triangle	circle	triangle

When my mom drives me to school I like to look out the window and find different shapes. On my corner is a red STOP sign, which is an octagon. On the next street there is a traffic light made up of a rectangle with one red, one green, and one yellow circle. The white speed limit signs are also rectangles. There is a red triangle sign where we go from the main road and a brown square sign that tell us that the name of the park is Brookside Park. Outside school is a yellow pentagon that shows people where the school crosswalk is. Our crossing guard, Mrs. Sylvan, holds a red octagon STOP sign, too.

Besides the many quadrilateral signs in store windows and traffic signs, I also see many other shapes while I ride to school each day. There is a house being built on our street, and the rafters have right triangles and isosceles triangles in them. There are equilateral triangles and trapezoids in the supports of the railroad bridge over the river. When I mentioned this assignment to my mom, I said I was missing a few shapes, which she helped me find. She pointed out a rhombus in the sign on the jewelry store that advertised their diamond sale. She also helped me see how the drawings of cubes in the kindergarten class windows are hexagons if you look at the lines of the cubes. And, finally, she helped me see the parallelograms that are also hidden in the support structure of the railroad bridge.

MORE THAN A TOOL, WHAT'S IN A CHAPTER?

In organizing the book around styles, we feel we can make the important goals of differentiating instruction and engaging students much more manageable for teachers of mathematics. This is why each chapter starts with a Math Tools Matrix that lays out the tools within the chapter and provides the reader with an at-a-glance overview (or "Quick Summary") of each tool. Open to any of the Math Tools Matrices contained in this new edition. Go ahead, pick your favorite one (Chapter 1: Mastery, pp. 2–3; Chapter 2: Understanding, pp. 52–53; Chapter 3: Self-Expressive, 106–107; Chapter 4: Interpersonal, pp. 154–155; Chapter 5: Mosaic, pp. 202–203). Notice that the tools are listed alphabetically and described down the left side of the matrix. If you follow the top row across the two-page spread, from left to right, you'll also notice that the columns are broken up into three distinct sections: "Episodes of Effective Instruction," "Connections (Sidebars)," and "Common Core State Standards—Standards for Mathematical Practice." By tracking a tool across the matrix, you can quickly collect the information you need about a tool to help you determine how well it fits your purposes.

HOW CAN I USE THESE TOOLS IN MY CLASSROOM?

Above all else, the second edition of *Math Tools* is a practical text—it's a guide that teachers of mathematics can use on the spot, from which they can select tools to design lessons, or that they can consult as a quick reference. Before taking the plunge into the math tools that make up the balance of this book, here are five different ways you can put these tools into effective practice:

1. ***Try one out.*** All of the tools in this book can help make a difference in the mathematics classroom. By our standards, a tool is only a tool if it has been proven effective in the classroom, addresses one or more of the Common Core Standards for Mathematical Practice, and plays an important role in making mathematics lessons come to life. So, pick a tool. Any tool. Try one out and watch what happens. Then try a few more. And be sure to try some from each style (or chapter). Before you know it, you and your students will have your own personal favorites, and a new teaching and learning dynamic will be in full swing in your classroom.

2. ***Use tools to help you build a practice or meet an objective.*** Looking for ways to help students make sense of problems and persevere in solving them? Just use the Math Tools Matrices that lead off each chapter to find one of the many tools that support the first Standard for Mathematical Practice (MP 1). Or maybe you're looking for a tool that will help you formatively assess student learning? Use the same chapter matrices to locate any number of tools with a Formative Assessment Connection sidebar. The tools in this book can help you and your students to build key mathematical practices and fulfill lesson or unit objectives in your classroom, even as they shift or evolve throughout the year.

3. ***Individualize instruction.*** Remember that the tools in this book are organized according to the different styles of mathematical thinking and learning they

engage. For example: Imagine you're working with a highly creative student (strong preference for Self-Expressive style) who just can't seem to memorize and follow the steps in a critical problem-solving procedure. Consider trying a tool like Math Recipe (see p. 130), which draws on the procedural nature of culinary arts to help students create a "recipe card" that outlines the steps in a mathematical procedure. Style-based individualization works because every style of learner has identifiable patterns of strength and weakness that can be accommodated or challenged. As Robert Sternberg (2006) has shown, allowing students of mathematics to think and work in their strong styles gives them a much better chance at mastering key content and skills. Even better, Sternberg's research showed that when students' preferences are accommodated, they become more likely to try to stretch as learners. You can use tools to teach to students' strengths or preferences as well as to challenge them to think and work in styles that they might otherwise avoid.

4. ***Differentiate instruction and increase engagement for the entire class.*** While personalizing mathematics instruction on the individual level is a powerful teaching and learning model in theory (and in practice for individual interventions), the truth is that most teachers of mathematics rarely have the luxury of working like doctors who see their patients one at a time. Since most classrooms are commonly organized by age or schedule availability, they will undoubtedly bring together groups of students with different abilities, experiences, prior knowledge, and conceptions of mathematics as a discipline. So, as a teacher of mathematics, how can you use math tools to engage all of your students and optimize their learning?

By engaging different styles of learners, math tools make the work of differentiating instruction and assessment for every learner a much more manageable proposition. All you need to do is rotate the tools you use in your classroom from each style (or chapter) to ensure that

- Mastery learners are getting the routine and direction they thrive on while they develop their ability to think conceptually and creatively.
- Understanding learners have the opportunity to think logically and independently while growing their capacities as thoughtful team members.
- Self-Expressive learners get the chance to use their imaginations while learning how to manage and master mathematical procedures.
- Interpersonal learners can learn as part of a problem-solving community, where mathematics connects strongly to the real world, while they build their critical reasoning skills.

Also, don't forget about the mosaic or multi-style tools in Chapter 5 (pp. 201–231), which are designed to engage all styles of learners simultaneously (e.g., Task Rotation, p. 224).

5. ***Design more powerful lessons and units.*** Tools work very well on their own, but they can also serve as "instructional building blocks" that teachers of mathematics use to construct powerful lessons and units. One of the most efficient and thoughtful design models follows this series of five episodes (Silver & Perini, 2010; Silver Strong & Associates, 2011):

Figure E Five Episodes of Effective Instruction

	Preparing Students for New Learning How do you establish your purpose, activate students' prior knowledge, and prepare students for learning?	
Deepening and Reinforcing Learning How do you help students solidify their understanding and practice new skills?	**Presenting New Learning** How do you present new information and provide opportunities for students to actively engage with content?	**Reflecting on and Celebrating Learning** How do you help students look back on their learning and refine their learning process?
	Applying Learning How do you help students demonstrate their learning and what kinds of evidence do you collect to assess student progress?	

Source: Silver Strong & Associates, 2011.

These episodes represent a synthesis of the preeminent instructional design models (Hunter, 1984; Marzano, 2007; Wiggins & McTighe, 2005), an evolution from the "Blueprint Model" of unit design featured in Silver and Perini's (2010) *Classroom Curriculum Design: How Strategic Units Improve Instruction and Engage Students in Meaningful Learning.* This is why we've taken care to identify which instructional episodes—or lesson/unit design purposes—each math tool supports. Just open up to any entry on any Math Tools Matrix. This element of the matrix allows teachers to quickly select and implement a math tool in their classroom at any point in their instructional sequence.

To concretize this point, let's look at an example of how a sixth-grade teacher designed a lesson on dividing fractions (see Figure F). Notice how she has selected math tools for each instructional episode to build a comprehensive lesson. For this particular sequence, this teacher is working toward a specific content standard on dividing fractions:

> **6.NS.1:** *Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.* (NGA Center, CCSSO, 2010b, p. 42)
>
> *Note:* 6.NS.1 is the Common Core coding that represents grade six (6), the Number System (NS) domain, standard one (1).

By integrating seven different math tools throughout five instructional episodes, the teacher has designed a lesson that will support the development of all eight Standards for Mathematical Practice.

Figure F Sample Lesson Design Notes

Preparing Students for New Learning • I will start with an Essential Question: How is dividing fractions related to dividing whole numbers? When do we divide fractions in the real world? • Then, I will transition to an engaging Hook: We're used to dividing whole numbers by fractions, but what happens when we divide fractions by fractions? Can you think of any examples of this in the real world? (Where in the World?, p. 193) • Before starting the lesson, I will give students a Pre-Assessment: They will use Range Finder (p. 181) to assess their current level of proficiency in dividing fractions by fractions.
Presenting New Learning • While I model and break down sample problems at the whiteboard, students will use Procedural Notes (p. 35) to record steps, reasons for each step, and line-by-line solutions. • Students will then work with a partner to apply division of fractions to a variety of scenarios, including sharing a Hershey's chocolate bar with fellow students. (Paired Learner, p. 174)
Deepening and Reinforcing Learning • To deepen students' learning while formatively assessing their progress, I will have students complete a series of Convergence Mastery (p. 4) quizzes and coaching sessions.
Applying Learning For an application task, I'll have students complete these four Task Rotation (p. 224) activities in order: 1. Mastery Task: Apply the division of fractions algorithm to these five problems of the form $\frac{a}{b} \div \frac{c}{d}$. 2. Interpersonal Task: Work with a partner. Review each other's work in the Mastery task, then divide the fractional values of the ingredients by 3. 3. Understanding Task: Write a paragraph that explains why the "keep it, change it, flip it" rule works. 4. Self-Expressive Task: Create your own algorithm for dividing fractions using common denominators.
Reflecting on and Celebrating Learning • As a final reflection activity, students will use No Uncertain Terms (p. 171) to write about what they've learned throughout the lesson.

In using tools across a sequence of five instructional episodes, teachers of mathematics can think their way through the bigger picture of lesson planning and unit design.

By choosing to implement tools from different chapters of this book, you can naturally accommodate and challenge all styles of learners, increase student motivation, and make your mathematics classroom more engaging. However, we also developed this enhanced edition of *Math Tools* to provide all mathematics teachers with a repertoire of high-impact instructional techniques that they can begin using tomorrow—techniques that are supported by research, that have been proved effective in mathematics classrooms, that build students' mathematical reasoning and thinking, and that can meet a variety of instructional objectives, thus leading to more cohesive lessons and units.