Workshop 1

Planning Math Lessons That Reach All Learners

Workshop Overview	6
Worksheet 1A: Workshop Participant Bingo	7
Arranging 12 Cubes Chart	8
Adapted Student Worksheet: Arranging Chairs	9
Worksheet 1B: Hands-On Exploration of the Math Activity	10
Worksheet 1C: Reactions to the Hands-On Exploration of the Math Activity	11
Worksheet 1D: Accessible Lesson Planning Chart	12
Worksheet 1E: Learning Goals	15
Worksheet 1F: Teaching Practices	16
Lesson Materials	17
Unit Guide for "Arranging Chairs" Lesson	17
Lesson Guide for "Arranging Chairs" Lesson	28
Student Work Samples	36
Homework Assignment	40
Demands of the Task Chart	41
Observation Chart	43
Worksheet 1G: Learning Goals	45
Reflection Questions	46

WORKSHOP 1 WORKSHOP OVERVIEW

This workshop will introduce participants to a neurodevelopmental framework and a case lesson on pre-algebra. The case is a third-grade lesson in which students use interlocking cubes to determine the factors of a number and examine patterns in factor families. Participants will use the neurodevelopmental framework to explore the mathematical demands of the case lesson. You will use video to observe the work done by Jashandeep, a student who has difficulties with organizing her thoughts and her work on paper. You will also reflect on instructional strategies to support Jashandeep and other students in the classroom who have different strengths and needs.

Participants will select a focal child from their own classroom and work with the members of their team to plan for an observation of this child, which will be guided by the neurodevelopmental framework and carried out before the next workshop session. Participants will record their observations, reflect on them, and share their work at the beginning of Workshop 2.

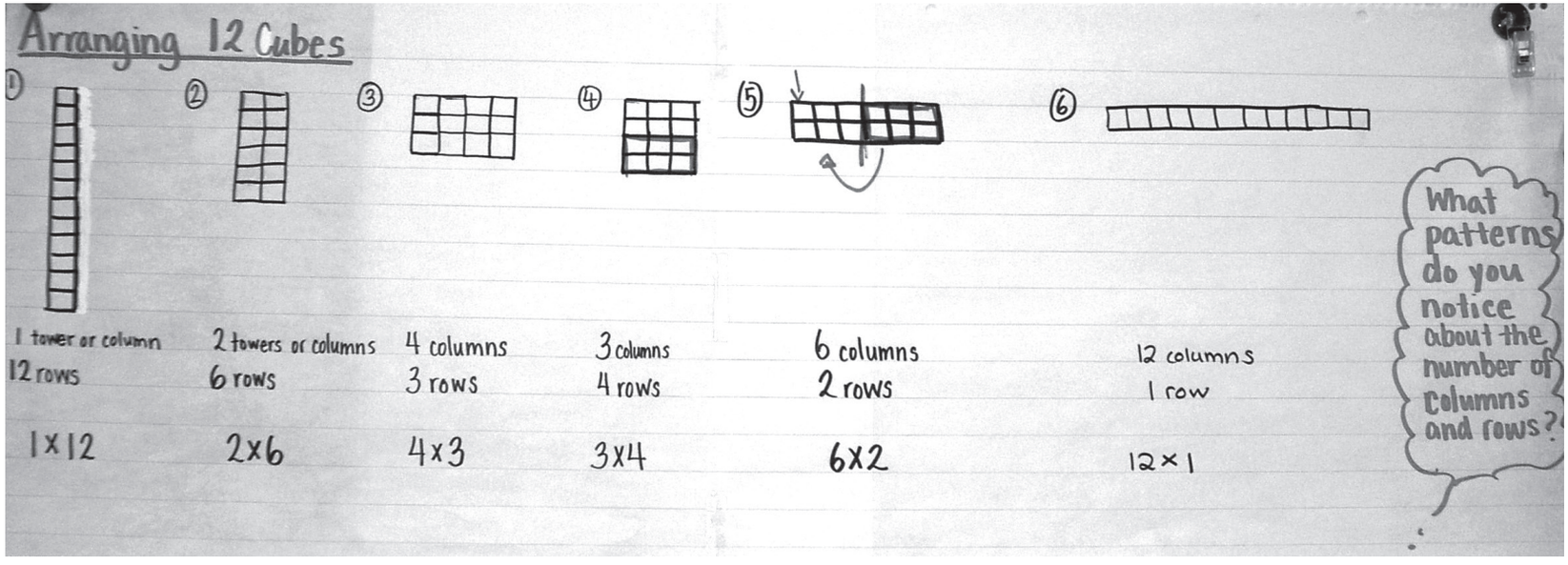
You will:

1. Learn how to analyze the demands of a mathematical task using the neurodevelopmental framework.
2. Learn how to use the neurodevelopmental framework to guide their observation of students' strengths and needs.
3. Enhance their understanding of instructional strategies that support students with strengths and needs in different neurodevelopmental functions.

WORKSHOP 1 **WORKSHEET 1A: WORKSHOP PARTICIPANT BINGO**

Introduce yourself to your fellow workshop participants and find out who meets the descriptors below. If you find a person who meets one of the descriptors, write his or her name in that cell. Do not list the same person in more than one cell.

Has a new home	Likes chocolate	Is an auditory learner	Has two children	Likes to cook
Likes to travel	Has a cat	Likes to dance	Speaks another language	Has a dog
Has a brother	Likes to swim	Is an only child	Sings in a choir	Likes coffee
Has one child	Likes to ride a bicycle	Is a vegetarian	Is a tactile/ kinesthetic learner	Plays the guitar
Likes to play baseball	Plays the piano	Plays tennis	Likes to snowboard	Is a visual learner



Created by Cindy Wang.

WORKSHOP 1 **ADAPTED STUDENT WORKSHEET**

Name(s): _____ Date: _____

Arranging Chairs

- ☐ 1. Explore one multiple. Find as many different arrangements as you can.
- ☐ 2. Record your arrangements/arrays with your partner.
- ☐ 3. What patterns are you and your partner noticing between the number of columns and rows?
- ☐ 4. How are the factors (or the number of columns and rows) changing?
- ☐ 5. Check your idea with another multiple. Does your pattern always work? (Use the bag or choose a multiple from 4 to 30)

WORKSHOP 1 **WORKSHEET 1B: HANDS-ON EXPLORATION OF THE MATH ACTIVITY**

Watch the video with the teacher's instructions for the activity. Carry out the activity with your group, following the teacher's instructions.

Observe and reflect on what skills you need to use to carry out this activity. Please use the space below to jot down your observations/reflections.

WORKSHOP 1 | **WORKSHEET 1C: REACTIONS TO THE HANDS-ON EXPLORATION OF THE MATH ACTIVITY**

Name: _____

Name of the Lesson: _____

Some things I found interesting when I carried out the activity myself:

1.

2.

3.

Some things I wouldn't have known about the activity if I hadn't explored it hands-on myself:

1.

2.

3.

WORKSHOP 1 **WORKSHEET 1D: ACCESSIBLE LESSON PLANNING CHART**

Name of Activity or Lesson Explored: *Arranging Chairs*

Focal Student Name: *Jashandeep*

Learning Areas (Barringer, Pohlman, & Robinson, 2010; Levine, 2002; Pohlman,2008)	What roles do these learning areas play in the Arranging Chairs activity?	How does Jashandeep respond to the demands of the task? Please note strengths and needs below.	How did Cindy Wang change the Arranging Chairs activity and what teaching practices does she use to make it more accessible to Jashandeep? What additional change would you make?
Higher Thinking • using and forming concepts • solving problems • logical thinking • creative and critical thinking			
Language • understanding mathematical language • using language to communicate with others and to clarify one's ideas			

Spatial Ordering • interpreting relationships within and between spatial patterns • organizing things in space • reasoning with images			
Sequential Ordering • organizing information in sequence • following directions • managing time			
Memory • short-term memory • active working memory • long-term memory			

Attention • controlling mental energy • maintaining focus • self-monitoring			
Psychosocial • using and understanding social language • collaboration • conflict resolution			
Motor Coordination • gross motor functions • fine motor functions • grapho-motor functions			

WORKSHOP 1 **WORKSHEET 1E: LEARNING GOALS**

Your Name:_____ Focal Student Pseudonym:_____

Name of the Lesson: _____

Review the introductory pages for the unit of the Arranging Chairs lesson to help you answer the questions below. The unit overview is found on pages 17 (1.12) to 27 (1.22). The lesson guide is on pages 28 (1.23) to 35 (1.30).

1. What are the learning goals of the lesson?

2. In what ways do you think this lesson connects to what students have studied in math before (this school year and before)?

3. How do you think what students learn in this lesson will help them with the math they will learn in the future (this school year and beyond)?

WORKSHOP 1 **WORKSHEET 1F: TEACHING PRACTICES**

Think about your focal child (or another child from your classroom). Which of these teaching practices might work for him or her? How would you use these practices?

Teaching Practices	How would you use these practices with your focal student and other students in your classroom?
Use concept mapping.	
Model problem-solving steps and approaches.	
Model critical thinking steps and approaches.	
Have students record and/or represent the steps they went through to solve the problem or analyze an idea.	
Have students work in mixed-ability groups.	
Post a written or pictorial chart that shows the steps for solving problems or for critical thinking.	
Encourage students to model a problem using diagrams and manipulatives.	
Use problems that are relevant to students' experiences and interests.	
Allow and encourage the use of calculators.	
Use graphic organizers to help students organize information and detect patterns so that they can more readily come up with a rule.	

WORKSHOP 1 **LESSON MATERIALS**

Unit Guide for "Arranging Chairs" Lesson

UNIT OVERVIEW

Things That Come in Groups

Content of This Unit To develop experience with some uses of multiplication and division, students work with things that come in groups, with patterns in the multiplication tables using 100 charts, and with rectangular arrays. They invent and solve problems about the number of legs on living creatures. Students become familiar with the multiplication tables up to the 12's, with emphasis on multiples with totals under 50. They also invent their own ways of solving multiplication and division problems.

Connections with Other Units If you are doing the full-year *Investigations* curriculum in the suggested sequence for grade 3, this is the second of 10 units. This unit introduces a variety of activities that use small numbers; these activities will be repeated later in *Landmarks in the Hundreds*, using larger numbers—with factors and multiples of 100.

This unit can be used successfully at either grade 3 or grade 4 as an introduction to multiplication and division, depending on the previous experience and needs of your students. Many of the ideas and games in this unit are picked up and extended in the grade 4 Multiplication and Division units.

Investigations Curriculum ■ Suggested Grade 3 Sequence

Mathematical Thinking at Grade 3 (Introduction)

▶ *Things That Come in Groups* (Multiplication and Division)

Flips, Turns, and Area (2-D Geometry)

From Paces to Feet (Measuring and Data)

Landmarks in the Hundreds (The Number System)

Up and Down the Number Line (Changes)

Combining and Comparing (Addition and Subtraction)

Turtle Paths (2-D Geometry)

Fair Shares (Fractions)

Exploring Solids and Boxes (3-D Geometry)

WORKSHOP 1 **LESSON MATERIALS**

Investigation 1 ■ Things That Come in Groups

Class Sessions	Activities	Pacing
Session 1 (p. 4) MANY THINGS COME IN GROUPS	Naming Things That Come in Groups Asking Multiplication Questions Brainstorming About Groups Homework: Things That Come in Groups	minimum 1 hr
Session 2 (p. 7) HOW MANY IN SEVERAL GROUPS?	Pictures of Things That Come in Groups Writing "Groups of" as Multiplication Homework: Pictures of Things That Come in Groups	minimum 1 hr
Session 3 (p. 12) WRITING AND SOLVING RIDDLES	Writing Riddles for Our Pictures Teacher Checkpoint: Do They Understand Multiplication? Homework: Riddled with Riddles!	minimum 1 hr
Session 4 (Excursion)* (p. 16) EACH ORANGE HAD 8 SLICES	How Many Altogether?	minimum 1 hr
Ten-Minute Math ■ Counting Around the Class		

*Excursions can be omitted without harming the integrity or continuity of the unit, but they offer good mathematical work if you have time to include them.

Mathematical Emphasis

- Finding things that come in groups
- Using multiplication notation
- Using multiplication to mean groups of groups
- Writing and illustrating multiplication sentences

Assessment Resources

What About Notation? (Teacher Note, p. 11)

Teacher Checkpoint: Do They Understand Multiplication? (p. 14)

The Relationship Between Division and Multiplication (Teacher Note, p. 15)

How Many Petals? How Many Bugs? (Dialogue Box, p. 19)

Materials

Calculators

Snap™ Cubes

Art materials: paper; colored pencils, markers, or crayons

Large paper

Each Orange Had 8 Slices by Paul Giganti, Jr., and Donald Crews (opt.)

Scissors

Tape

Student Sheets 1–3

Family letter

WORKSHOP 1 **LESSON MATERIALS**

Investigation 2 ▪ Skip Counting and 100 Charts

Class Sessions	Activities	Pacing
Session 1 (p. 22) HIGHLIGHTING MULTIPLES IN 100 CHARTS	Highlighting 2's and 3's Making Books of 100 Charts Homework: Multiples on 100 Charts	minimum 1 hr
Session 2 (p. 26) USING THE CALCULATOR TO SKIP COUNT	Skip Counting 4's and More Homework: More Multiples on 100 Charts	minimum 1 hr
Sessions 3 and 4 (p. 28) MORE PRACTICE WITH MULTIPLES	Choice Time: Exploring Multiples and Patterns Teacher Checkpoint: Using the Skip Counting Circles Homework: Silly Story Problems Homework: Patterns Across the Charts	minimum 2 hr
Sessions 5 and 6 (p. 33) DISCUSSING NUMBER PATTERNS	Patterns in Multiples of 9 and 11 Numbers That Appear on Two Charts Discussion: Patterns Across the Charts Playing Cover 50 Homework: Cover 50 Game	minimum 2 hr
Ten-Minute Math ▪ Counting Around the Class		

Mathematical Emphasis

- Recognizing that skip counting represents multiples of the same number and has a connection to multiplication
- Looking for patterns in multiples of 2 through 12 on the 100 chart
- Understanding that the patterns numbers make can help us multiply those numbers

Assessment Resources

Students' Problems with Skip Counting (Teacher Note, p. 25)

Teacher Checkpoint: Using the Skip Counting Circles (p. 31)

Multiples of 6 (Dialogue Box, p. 39)

Cover 50 (Dialogue Box, p. 39)

Materials

Overhead projector, transparencies, and transparency pens

Calculators

Snap™ Cubes

Art materials: colored pencils, markers, or crayons

Each Orange Had 8 Slices by Paul Giganti, Jr., and Donald Crews (opt.)

Envelopes or resealable plastic bags

Scissors

Stapler

Student Sheets 4–6

Teaching resource sheets

WORKSHOP 1 **LESSON MATERIALS**

Investigation 3 ■ Arrays and Skip Counting

Class Sessions	Activities	Pacing
Sessions 1 and 2 (p. 42) ARRANGING CHAIRS	Arranging Chairs in Rectangular Arrays Arranging More Chairs Making Array Cards Homework: Cutting Out Array Cards	minimum 2 hr
Sessions 3 and 4 (p. 48) ARRAY GAMES	Counting Squares in Arrays Playing Array Games Homework: Array Games Extension: What Number Has the Most Arrays?	minimum 2 hr
Session 5 (p. 53) THE SHAPES OF ARRAYS	Discussing Array Game Strategies Assessment: Arrays That Total 36 Homework: More Array Games	minimum 1 hr
Ten-Minute Math ■ Counting Around the Class		

Mathematical Emphasis

- Recognizing that finding the area of a rectangle is one situation where multiplication can be used
- Using arrays to skip count
- Using arrays with skip counting to multiply and divide
- Finding factor pairs
- Making connections between number and shape

Assessment Resources

Arranging Chairs (Dialogue Box, p. 47)

Assessment: Arrays That Total 36 (Teacher Note, p. 55)

Materials

Overhead projector, transparencies, and pen
Calculators
Snap™ Cubes
Resealable plastic bags
Scissors
Array Cards
Student Sheet 7
Teaching resource sheets

WORKSHOP 1 **LESSON MATERIALS**

Investigation 4 ■ The Language of Multiplication and Division

Class Sessions	Activities	Pacing
Sessions 1 and 2 (p. 58) MULTIPLY OR DIVIDE?	Solving Story Problems Acting Out Number Sentences Different Ways to Write Problems Writing Multiplication and Division Sentences Teacher Checkpoint: Do They Understand the Notation? Homework: More Story Problems Homework: Decribe the Problem Extension: Interpreting Problems on Standardized Tests	minimum 2 hr
Sessions 3 and 4 (p. 70) WRITING AND SOLVING STORY PROBLEMS	Writing Story Problems A Class Book of Problems Solving Problems in the Class Book Homework: The Class Book at Home Extension: Problems About the Class	minimum 2 hr
Ten-Minute Math ■ Likely or Unlikely?		

Mathematical Emphasis

- Understanding relationships between multiplication and division
- Identifying whether word problems can be solved using division and/or multiplication
- Using multiplication and division notation to write number sentences

Assessment Resources

Teacher Checkpoint: Do They Understand the Notation? (p. 66)

Talking and Writing About Division (Teacher Note, p. 67)

Would You Use Multiplication or Division? (Dialogue Box, p. 68)

Two Kinds of Division: Sharing and Partitioning (Teacher Note, p. 68)

Materials

Overhead projector, transparencies, and pen

Calculators

Snap™ Cubes

Art materials: colored paper; plain paper; colored pencils, markers, or crayons

Chart paper and marker

Stapler

Student Sheets 8–11

Teaching resource sheets

WORKSHOP 1 **LESSON MATERIALS**

Investigation 5 ■ Problems with Larger Numbers

Class Sessions	Activities	Pacing
Session 1 (p. 76) CALCULATING SAVINGS	How Much Would You Save? Homework: How Much Would You Save? Extension: How Many Months Old Are You?	minimum 1 hr
Session 2 (p. 79) MANY, MANY LEGS	Discussion: What Could We Buy? How Many Legs? Planning a Survey Homework: Creatures in Our Homes and Neighborhoods	minimum 1 hr
Session 3 (p. 83) DATA TABLES AND LINE PLOTS	Expanding Our Data Tables Making a Line Plot Problems from Our Own Data Homework: Finishing Display Pages	minimum 1 hr
Session 4 (p. 87) A RIDDLE WITH 22 LEGS	Assessment: A Riddle with 22 Legs Choosing Student Work to Save	minimum 1 hr
Ten-Minute Math ■ Likely or Unlikely?		

Mathematical Emphasis

- Multiplying and dividing in real-life situations and using patterns to solve multiplication and division problems
- Organizing and presenting data in tables and line plots
- Sorting out complex problems that require both multiplication and addition
- Making up division and multiplication story problems from real data

Assessment Resources

Assessment: A Riddle with 22 Legs (p. 87)

Choosing Student Work to Save (p. 89)

Materials

Overhead projector, transparencies, and pen

Calculators

Art materials: drawing paper; colored pencils, markers, or crayons

Student Sheets 12–15

Teaching resource sheets

WORKSHOP 1 LESSON MATERIALS

MATERIALS LIST

Following are the basic materials needed for the activities in this unit.

- Snap™ Cubes (interlocking cubes): 50 per student
- Array Cards (manufactured, or use blackline masters at the back of this book to make your own sets). Cutting out Array Cards can take students a great deal of time. If you use the blackline masters, enlist the help of classroom aides to cut out (but not label) sets of cards for your students to use in class. Since students will need a set for homework, they can also take a set home to cut out with the help of family members. Array Cards are introduced in Investigation 3.
- Calculators: at least 1 per pair of students
- *Each Orange Had 8 Slices* by Paul Giganti, Jr., and Donald Crews (optional)
- Scissors: 1 per student
- Legal-size envelopes
- Quart-size resealable plastic bags
- Large paper for making class lists
- Colored paper, drawing paper
- Colored pencils, markers, or crayons
- Overhead projector
- Blank overhead transparencies, pens
- Scissors
- Stapler
- Chart paper, marker

The following materials are provided at the end of this unit as blackline masters. A Student Activity Booklet containing all student sheets and teacher resources needed for individual work is available.

Family Letter (p. 98)
Student Sheets 1–15 (p. 99)
Teaching Resources:
- Cover 50 Game (p. 105)
- How to Play Cover 50 (p. 106)
- Array Cards (p. 108)
- How to Make Array Cards (p. 114)
- The Arranging Chairs Puzzle (p. 115)
- How to Play Multiplication Pairs (p. 116)
- How to Play Count and Compare (p. 117)
- Number Problems (p. 122)
- Half-Inch Graph Paper (p. 127)

Practice Pages (p. 129)

Related Children's Literature

Axelrod, Amy. *Pigs Will Be Pigs.* New York: Four Winds Press, 1994.

Carle, Eric. *The Very Hungry Caterpillar.* New York: World Publishing, 1969.

Giganti, Paul, Jr., and Donald Crews. *Each Orange Had 8 Slices.* New York: Greenwillow, 1992.

Low, Joseph. *Mice Twice.* New York: Atheneum, 1980.

Note: The blackline masters for this unit are not included in this book.

ABOUT THE MATHEMATICS IN THIS UNIT

In this unit, students develop their own strategies for doing multiplication and division problems. They discover that both types of problems deal with equal groups, but each will answer different questions about the groups. Multiplication is typically used when the size of each group and the number of groups is known, and we want to find the total number of items. Division is most often used when the total quantity is known, and we want to find out either the number or the size of the groups.

As students develop strategies to use in multiplication and division situations, it is critical that they develop visual images that support their work. They may use an array of squares, for example, to visualize an important multiplication relationship—that the solution to 7×6 is the same as the solution to 6×7. As students skip count on a 100 chart, they begin to recognize characteristics of particular multiples. They will see, for example, that all the multiples of 2, 4, and 6 are even numbers, or that all the multiples of 5 end in either 5 or 0. Students may at first visualize multiplication as repeated addition, since this process is more familiar to them.

Throughout this unit, it is most important to support students' efforts to make sense out of multiplication and division. As students develop their own strategies, they are aided by knowing many of the single-digit multiplication pairs. We do not expect them to memorize all the multiples, but as they look at patterns in the tables and construct the multiples again and again by skip counting, students will commit many of them to memory. They will also pick up ways to solve others quickly—for example, by using a known answer to find an unknown one ("8×6 is like 4×6 twice, so it's 24 and 24, and that's 48").

Students also learn to read standard multiplication and division notation and to use this notation to record their work. They must also learn that notation communicates the problem to be solved but doesn't prescribe the method of solution.

When students see problems written in standard forms such as these:

$$\begin{array}{r} 56 \\ \times\ 8 \\ \hline \end{array} \qquad 4\overline{)132}$$

the form of the problem may trigger use of poorly understood, and often inefficient, algorithms. For example, in the first problem, students might start to say, "8 times 6 is 48, put down the 8 and carry the 4...." This procedure obscures the use of good number sense and often leads students to fragment a number into its digits and lose track of the quantities represented by the numerals. Good mental strategies often start from the left, focusing first on the largest part of the number, rather than the smallest: "eight 50's is 400, eight 6's is 48, so that's 448."

Students need to develop efficient computation strategies, many of which will be mental strategies, but these must be based on their understanding of the quantities and their relationships, not on memorized procedures. We would like students to recognize multiplication and division problems written in all of the notations they are likely to see in elementary school but to solve them in their own way.

At the beginning of each investigation, the Mathematical Emphasis section tells you what is most important for students to learn about during that investigation. Many of these mathematical understandings are difficult and complex. Students gradually learn more and more about each idea over many years of schooling. Individual students will begin and end the unit with different levels of knowledge and skill, but all will gain greater knowledge about multiples and factors and about some meanings and notation for multiplication and division.

WORKSHOP 1 LESSON MATERIALS

ABOUT THE ASSESSMENT IN THIS UNIT

Throughout the *Investigations* curriculum, there are many opportunities for ongoing daily assessment as you observe, listen to, and interact with students at work. In this unit, you will find three Teacher Checkpoints:

Investigation 1, Session 3:
Do They Understand Multiplication? (p. 14)

Investigation 2, Sessions 3–4:
Using the Skip Counting Circles (p. 31)

Investigation 4, Sessions 1–2:
Do They Understand the Notation? (p. 66)

This unit also has two embedded Assessment activities:

Investigation 3, Session 5:
Arrays That Total 36 (p. 54)

Investigation 5, Session 4:
A Riddle with 22 Legs (p. 87)

In addition, you can use almost any activity in this unit to assess your students' needs and strengths. Listed below are questions to help you focus your observations in each investigation. You may want to keep track of your observations for each student to help you plan your curriculum and monitor students' growth. Suggestions for documenting student growth can be found in the section About Assessment.

Investigation 1: Things That Come in Groups

- How easily do students generate ideas of items that come in groups? Where do they look for ideas? (For example, do they check around the classroom? Around school? At home? At a store?)
- How do students write number sentences when describing groups of objects? How do they interpret standard notation? How do they solve problems presented in standard notation?
- How do children show that they understand the structure of multiplication problems? How do they explain the meaning of a multiplication equation? Do they talk about groups?
- How do children write and interpret multiplication sentences? How do they draw an illustration to represent a multiplication sentence? Do they understand the connection between the sentence and the illustration?

Investigation 2: Skip Counting and 100 Charts

- How do students skip count during various activities? What numbers do they seem most comfortable skip counting with? How do they use skip counting and the 100 chart to solve multiplication problems? What language do students use to discuss factors and multiples?
- What patterns do students notice about the numbers that are (and are not) highlighted on their multiple charts? What kinds of observations do they make? Do they investigate the behavior of odd and even numbers on the multiple charts?
- How do students use their 100 charts (and the patterns on them) to help them solve multiplication problems? What do students notice when overheads of two different factors of a multiple are overlaid? What predictions do they make about other multiples on the basis of this knowledge?

Investigation 3: Arrays and Skip Counting

- How do students find the total number of squares in an array? Do they see this as a multiplication situation? How do they use multiplication?
- When figuring the total of an array, how do the students count the squares that make up that array? Do they count by 1's, or do they skip count by the number in a row or column?
- How do students find one dimension of an array when they have the second dimension and the total number of squares?
- How do students go about finding all possible dimensions for array shapes? How do they use their knowledge of one factor pair to influence their choice of another (for example, 2×4 is the same as 4×2)? Are they organized and systematic in their approach?
- How do students identify the dimensions of an array when they know only the total number of squares in that array? How do they use what they know about relationships between shape and number?

Investigation 4: The Language of Multiplication and Division

- How do students express multiplication problems as division problems? How do they use related problems of the opposite operation to help them?
- How do children interpret word problems? How do they decide whether to use multiplication or division?
- How do children use notation to write the number sentences for word problems? How many types of notation do they use? Do their equations match the question the word problem is asking?

Investigation 5: Problems with Larger Numbers

- What methods do students use to multiply or divide in real-life situations? Do they use known patterns?
- How do students organize data? Are their representations clear? Can you gather information from them? How do they clarify their representations when someone has a question?
- How do children make sense of and solve problems involving both multiplication and addition? How do they organize and keep track of their work?
- How do students make up story problems from real data? What variables do children include? How do they make them challenging?

Assessment Sourcebook

In the *Assessment Sourcebook* you will find End-of-Unit Assessment Tasks and Assessment Masters available in English and Spanish. You will also find suggestions to help you observe and evaluate student work and checklists of mathematical emphases with space for you to record individual student information.

WORKSHOP 1 LESSON MATERIALS

PREVIEW FOR THE LINGUISTICALLY DIVERSE CLASSROOM

In the *Investigations* curriculum, mathematical vocabulary is introduced naturally during the activities. We don't ask students to learn definitions of new terms; rather, they come to understand such words as *factor* or *area* or *symmetry* by hearing them used frequently in discussion as they investigate new concepts. This approach is compatible with current theories of second-language acquisition, which emphasize the use of new vocabulary in meaningful contexts while students are actively involved with objects, pictures, and physical movement.

Listed below are some key words used in this unit that will not be new to most English speakers at this age level, but may be unfamiliar to students with limited English proficiency. You will want to spend additional time working on these words with your students who are learning English. If your students are working with a second-language teacher, you might enlist your colleague's aid in familiarizing students with these words, before and during this unit. In the classroom, look for opportunities for students to hear and use these words. Activities you can use to present the words are given in the appendix, Vocabulary Support for Second-Language Learners (p. 95).

question, statement, illustrate In Investigation 1, students write and *illustrate* their own multiplication "riddles," making two or more *statements* involving numbers, and ending with a *question*.

chart, row, column Students color in multiples on the 100 *chart* in Investigation 2, looking for visual patterns in its *rows* and *columns*, as well as diagonals.

calculator, press, equals key, plus key Students learn to use the calculator to skip count by any number, *pressing* that number, the *plus key*, and then the *equals key* repeatedly.

day, week, month, amount In a savings problem in Investigation 5, students find the *amount* of money they would save in a *week* or a *month* by saving the same amount each *day*.

creature, leg, -legged In a series of activities during Investigation 5 that involve working with larger numbers, students investigate various combinations of creatures with different numbers of legs.

Multicultural Extensions for All Students

Whenever possible, encourage students to share words, objects, customs, or any aspects of daily life from their own cultures and backgrounds that are relevant to the activities in this unit. For example:

- When students are thinking of things that come in groups during the first investigation, encourage them to include groups of things that may reflect their culture—such as the number of dancers in a particular dance, or the number of playing pieces in a popular game.
- When students are writing story problems in Investigation 4, encourage the use of culture-specific references (to items of food, for example) so that in sharing their problems with the class, they share a little of themselves as well.

Note: The appendix "Vocabulary Support for Second-Language Learners" is not included in this book.

INVESTIGATION 3

Arrays and Skip Counting

What Happens

Sessions 1 and 2: Arranging Chairs Challenged to find different ways to arrange rows of chairs for an audience, students manipulate 12 cubes to see how many different rectangles they can make. They list the dimensions of these rectangles and the factors of 12. Students work individually and in pairs to determine the factors of other numbers by making rectangles. They also begin making sets of array cards for use throughout the investigation.

Sessions 3 and 4: Array Games Students talk about ways to count the total in arrays, and they learn two array games—Multiplication Pairs, and Count and Compare. In addition, students can choose to do further work on the Arranging Chairs puzzle. These activities give students practice multiplying and dividing and encourage them to develop connections between number and shape.

Session 5: The Shapes of Arrays Students briefly discuss strategies for working with arrays. Then they do an assessment problem that involves identifying by shape the arrays with a total of 36 and identifying the factors of 36. Students continue to play array games, if time permits.

Mathematical Emphasis

- Recognizing that finding the area of a rectangle is one situation where multiplication can be used
- Using arrays to skip count
- Using arrays with skip counting to multiply and divide
- Finding factor pairs
- Making connections between number and shape

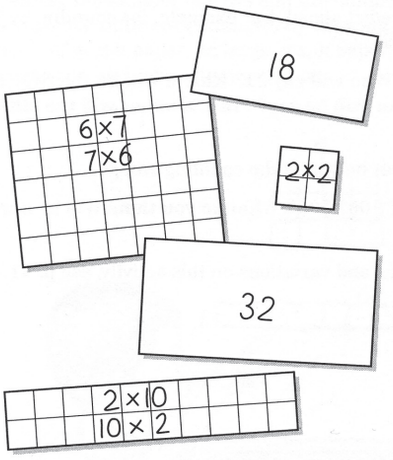
WORKSHOP 1 **LESSON GUIDE FOR "ARRANGING CHAIRS" LESSON**

INVESTIGATION 3

What to Plan Ahead of Time

Materials

- Overhead projector and transparency pen (Sessions 1–4)
- Interlocking cubes: at least 30 per student (Sessions 1–2, 5)
- Calculators: 1 per pair (Sessions 1–2)
- Quart-size resealable plastic bags to hold Array Cards: 1 per student (Sessions 1–2)
- Scissors: 1 per student (Sessions 1–2)
- Array Cards (Sessions 1–4). If you do not have manufactured cards, make your own; see Other Preparation.



Other Preparation

- Duplicate student sheets and teaching resources (located at the end of this unit) in the following quantities. If you have Student Activity Booklets, copy only the item marked with an asterisk.

For Sessions 1–2

Half-inch graph paper (p. 127): 2–3 sheets per student (optional), 1 transparency* (optional)

Array sheets 1–6 (p. 108): 1 set per student (homework)

How to Make Array Cards (p. 114): 1 per student (homework)

For Sessions 3–4

The Arranging Chairs Puzzle (p. 115): 1 per student (homework)

How to Play Multiplication Pairs (p. 116): 1 per student (homework)

How to Play Count and Compare (p. 117): 1 per student (homework)

For Session 5

Student Sheet 7, Arrays That Total 36 (p. 107): 1 per student

- If you do not have manufactured Array Cards for *Investigations* grade 3, use the blackline masters at the back of this book to make 1 set of cards per student for class use. (See What to Plan Ahead of Time for Investigation 1.) Students make another set for homework use. (Sessions 1–2)
- Make overhead transparencies of the Array Cards. Cut apart the 51 arrays, which represent the multiplication combinations of the factors 2 through 12 with totals up to 50. Do not label the dimensions or the total. (Sessions 3–4)

Note: The student sheets, teaching resources, and blackline masters for this unit are not included in this book.

Sessions 1 and 2

Arranging Chairs

Materials

- Overhead projector, transparency pen
- Interlocking cubes (30 per student)
- Calculators (1 per pair)
- Half-inch graph paper (2–3 sheets per student, 1 transparency) (optional)
- Array Card pages (1 set per student, homework)
- Scissors (1 per student)
- Quart-size resealable plastic bags (1 per student)
- How to Make Array Cards (1 per student, homework)

What Happens

Challenged to find different ways to arrange rows of chairs for an audience, students manipulate 12 cubes to see how many different rectangles they can make. They list the dimensions of these rectangles and the factors of 12. Students work individually and in pairs to determine the factors of other numbers by making rectangles. They also begin making sets of Array Cards for use throughout the investigation. Their work focuses on:

- making rectangles for quantities of 12 and other numbers
- finding factors of 12 and other numbers

Ten-Minute Math: Counting Around the Class Continue to do Counting Around the Class two or three times during this investigation. Remember that this activity is intended to be done outside of math time.

Count by numbers whose patterns are now reasonably familiar to your students: 2's, 5's, 10's, and perhaps 3's, 4's, or 9's. Students can refer to their highlighted charts if they wish.

Ask students to predict ahead. For example, for counting by 3's, ask questions like these:

Who will say 15? Who will say 21? Khanh will be the twelfth student. What number will he say? What number will the student after Khanh say?

Ask questions about how high the counting will go.

Will we reach 50? 100? 200? What do you think will be our final number?

For full instructions and variations on this activity, see p. 91.

WORKSHOP 1 **LESSON GUIDE FOR "ARRANGING CHAIRS" LESSON**

Activity

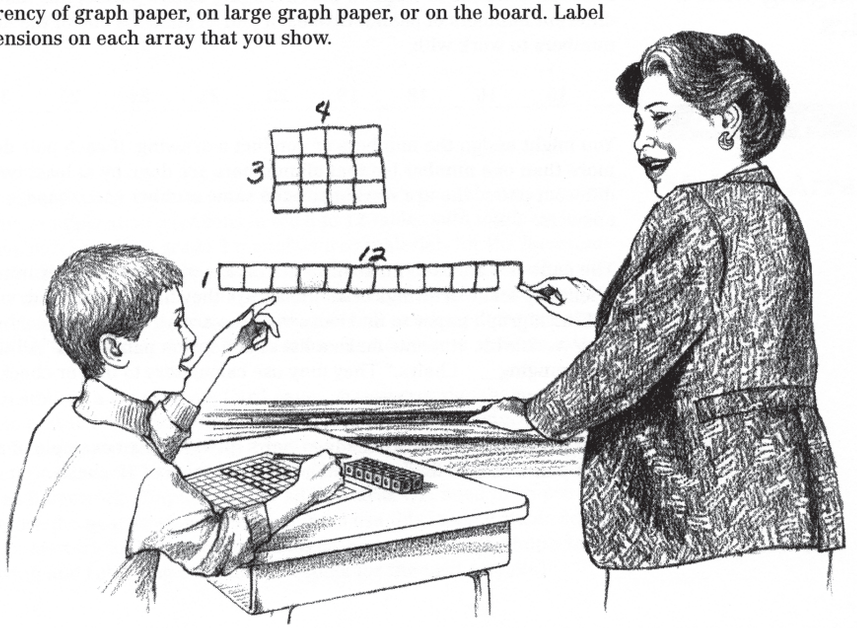
Arranging Chairs in Rectangular Arrays

Introducing Arrays Each student needs 12 cubes to work with. Put 12 cubes on the overhead projector. Briefly explain the task:

Here's a puzzle to solve. We'll call it the Arranging Chairs puzzle. Pretend these 12 cubes are chairs. You want to arrange them in straight rows for an audience to watch a class play. You need to arrange the chairs so that there will be the same number in every row. How many different ways could you do this? How many chairs would be in each row? How many rows would there be? Try many different ways to arrange the chairs, even if some ways seem a bit silly for watching a class play.

❖ **Tip for the Linguistically Diverse Classroom** To support your explanation of the task, model the arranging of four chairs in different ways—one row of 4 across, four rows of 1 (one behind another), and two rows of 2. Make the corresponding arrangement of cubes for each.

Students spend some time making as many different rectangles as they can using the 12 cubes. When they have made several possible arrays, ask them to identify the number of rows and the number of chairs in each row. Show the students' different rectangles by drawing them on an overhead transparency of graph paper, on large graph paper, or on the board. Label the dimensions on each array that you show.



WORKSHOP 1 LESSON GUIDE FOR "ARRANGING CHAIRS" LESSON

Identify for students the words *array* and *dimension*.

Mathematicians sometimes call things that are grouped this way to form a rectangle an *array*.

***Dimension* is a name for the length or width of a rectangle. What are the *dimensions* of your rectangles? See how I'm labeling the dimensions of the rectangles as I draw them, the *length* and the *width*.**

Use the term *by* when talking about dimensions and students will copy you; for example, "The dimensions of this rectangle are 2 *by* 6." List the pairs of dimensions on the board.

3×4	2×6	1×12
4×3	6×2	12×1

Have we made all of the possible rectangles? Is our list of dimensions complete? Each of the dimensions on this list is a *factor* of 12. What are all the factors of 12? (1, 2, 3, 4, 6, 12)

Activity

Arranging More Chairs

Students continue to work on the Arranging Chairs puzzle, this time with different numbers of chairs. Give each pair of students one of the following numbers to work with:

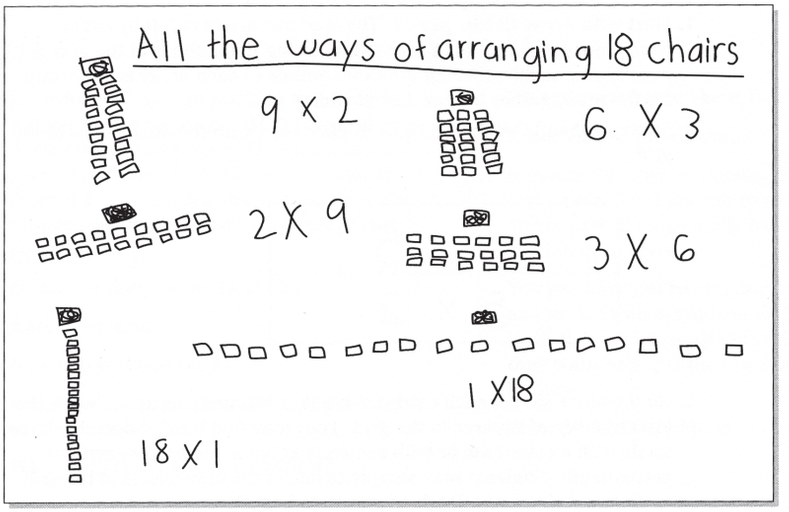
15 16 18 19 20 21 24 25 30

You might assign the numbers or conduct a drawing. If each pair does more than one number (so that all numbers are done by at least two pairs), different pairs who are working on the same number can compare their answers.

The pairs use cubes to make different arrays of chairs for their number. Then they make drawings of all the arrays they find. If you want, supply half-inch graph paper to make drawing the arrays easier. For each number they work with, students make a list of dimension pairs titled "All the Ways of Arranging ___ Chairs." They may use calculators to find or check the dimensions of their arrays.

See the **Dialogue Box**, Arranging Chairs (p. 47), for an example of a student pair trying to find all possible arrangements of 15 chairs.

WORKSHOP 1 **LESSON GUIDE FOR "ARRANGING CHAIRS" LESSON**



When they are finished, invite pairs of students to report their findings, one number at a time. Make a list of the dimensions of the arrays students made for each number. Point out that the number 19 makes only two arrays—1 by 19, and 19 by 1. Remind students about prime numbers—those that didn't turn up on any highlighted charts except their own. Ask:

What other numbers would have only two arrays?

Activity

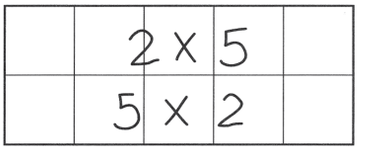
Making Array Cards

The six pages of Array Cards provide 51 arrays—every possible array representing the multiplication equations in the 2 to 12 tables *with totals up to 50.* If you have purchased the grade 3 manufactured materials for the *Investigations* curriculum, you will have printed sets of these 51 Array Cards that students can use in class. If not, you or an aide will have already made sets of cards for class use. In either case, each student will also benefit from making an individual set of paper Array Cards to use for homework assignments.

Give each student a set of Array Card pages, scissors, and a quart-size resealable plastic bag to hold the cut-apart array cards. Introduce the process of cutting out and labeling the cards as a whole-class activity. Give students time to practice with one or two sheets, and then have them do the rest as homework. Emphasize that all the cards will need to be prepared before the next session. (Some teachers use this student-made set for both homework and classwork. In that case, underscore the importance of preparing and returning the cards to school for the next session.)

Explain the procedure:

1. Start with Array Cards, page 1. The students are to carefully cut out each individual array on the sheet, following the outlines of the grid as exactly as possible. (Seeing the exact outline of each array is important for the array games they will be playing.)
2. Students then label the grid side of each card with the dimensions of the grid.



3. On the other side of each card (the blank side), students are to write the total number of squares in the grid. They may find it helpful to check the totals with a classmate or with someone at home before they write it permanently. Students may also write one of the dimensions of the grid on the total side, very lightly in pencil, to help them when the arrays are new. These can be erased when the students feel more confident.

4. Students write their initials on each card (in a corner away from the numbers) and store the labeled cards in the plastic bag.

Before students work independently, you might have them make some of the larger arrays. As students are working, walk around the room and observe whether they understand what to do and how each card should be prepared.

Sessions 1 and 2 Follow-Up

Homework

Cutting Out Array Cards For homework, students finish cutting out the Array Cards that they began making in class. Be sure to send home, besides the Array Card sheets, the plastic storage bags and copies of the sheet How to Make Array Cards as a reminder of how the cards are to be labeled. If this is their classroom set, emphasize that students are to bring their bags of cards back to class with them tomorrow. If this is a set to keep at home, remind students to store their bags of cards in a safe place.

WORKSHOP 1 **LESSON GUIDE FOR "ARRANGING CHAIRS" LESSON**

D I A L O G U E B O X

Arranging Chairs

Ricardo and Kate are working to find all the different ways to arrange 15 "chairs" for the Arranging Chairs puzzle. They have made 3 rows of 5, 5 rows of 3, 1 row of 15, and 15 rows of 1, as their drawing shows. They are now considering whether they have all the possibilities.

Is that all there is for 15 chairs?

Kate: I think so.

How can you find out?

Ricardo: From experimenting, but nothing is even any more.

What does that have to do with it?

Ricardo: I could do like 14. Let's see, 7 + 7 is 14, so 7 + 8 … but that wouldn't be even rows, so it has to be two odds or something.

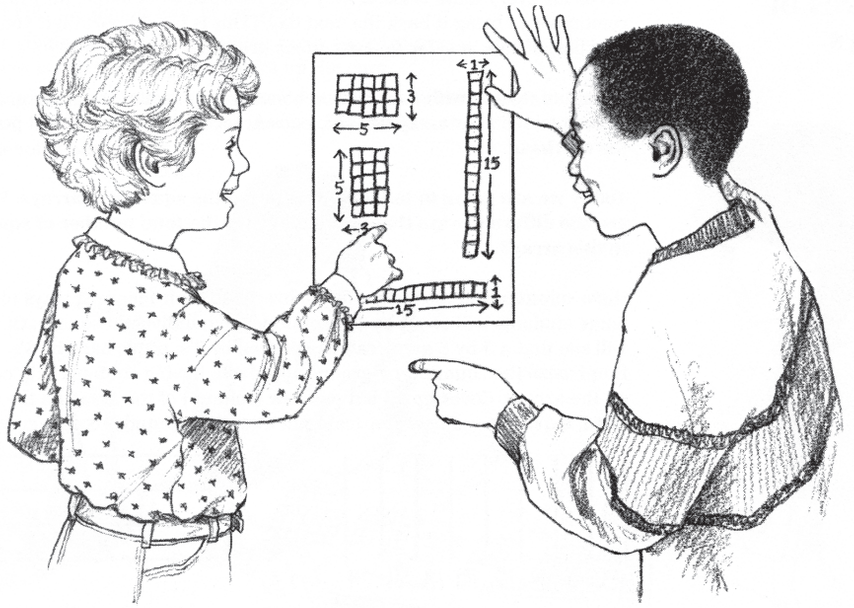
What do you know about two odds?

Kate: They make an even.

Ricardo: So, if we're missing any, it has to be odd + even. 1 + 2 doesn't work. 1 + 3 doesn't. I guess *none* of those work, because that wouldn't ever be even rows.

You see how you're starting at the beginning and making an organized list? [*Ricardo nods.*] **Well, that's what mathematicians do when they want to see if they've found all the possibilities.**

Ricardo: You're kidding!



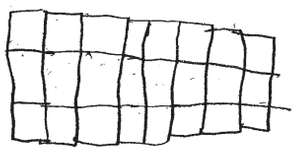
Sessions 1 and 2: Arranging Chairs ■ **47**

WORKSHOP 1 **STUDENT WORK SAMPLES**

Name(s): Jashandeep Siddiqe Date: 1/25/06

Arranging Chairs

- ☐ 1. Explore one multiple. Find as many different arrangements as you can.
- ☐ 2. Record your arrangements/arrays with your partner.
- ☐ 3. What patterns are you and your partner noticing between the number of columns and rows?
- ☐ 4. How are the factors (or the number of columns and rows) changing?
- ☐ 5. Check your idea with another multiple. Does your pattern always work? (Use the bag or choose a multiple from 4 to 30)

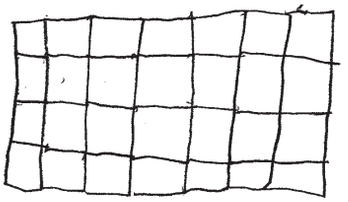


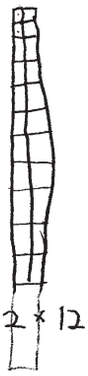
1 × 24 = 24

8 × 3 = 24

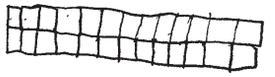
3 towers

8 Columns



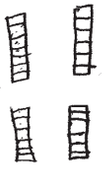


2 × 12 = 24



12 × 2 = 24

3 × 8 = 24



6 × 4 = 24

1 But when you have an odd or even you alone could brean number's that are even not odd.

WORKSHOP 1 **STUDENT WORK SAMPLES**

Name(s): David Taren Date: _____

Arranging Chairs

- ☐ 1. Explore one multiple. Find as many different arrangements as you can.
- ☐ 2. Record your arrangements/arrays with your partner.
- ☐ 3. What patterns are you and your partner noticing between the number of columns and rows?
- ☐ 4. How are the factors (or the number of columns and rows) changing?
- ☐ 5. Check your idea with another multiple. Does your pattern always work? (Use the bag or choose a multiple from 4 to 30)

column row
1 x 24 = 24
2 x 12 = 24

12 rows
12 colmaus

12 rows
2 colmaus 12 x 2 = 24

4 rows
6 colmaus 4 x 6 = 24

1 row
24 colmaus 1 x 24 = 24

4 rows 4 x 6 = 24

6 colmaus

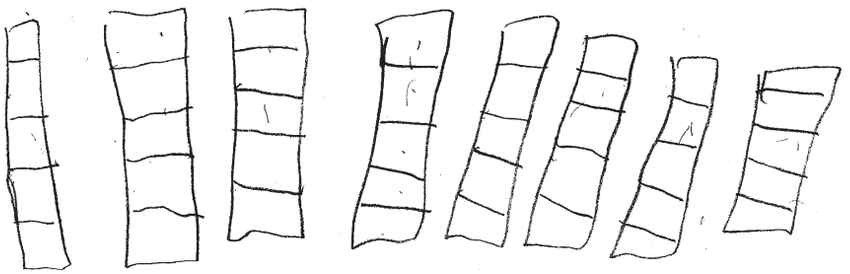
6 rows 6 x 4 = 24
colmaus

WORKSHOP 1 **STUDENT WORK SAMPLES**

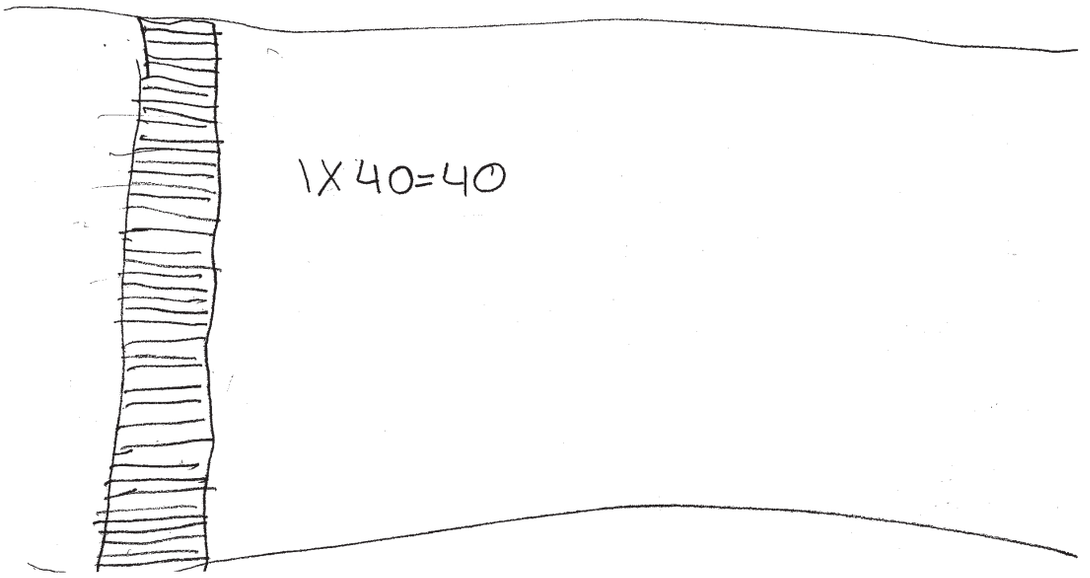
Name(s): Rahul, Edgar, Harpreet Date: 1/18/06

Arranging Chairs

- ☑ 1. Explore one multiple. Find as many different arrangements as you can.
- ☐ 2. Record your arrangements/arrays with your partner.
- ☐ 3. What patterns are you and your partner noticing between the number of columns and rows?
- ☐ 4. How are the factors (or the number of columns and rows) changing?
- ☐ 5. Check your idea with another multiple. Does your pattern always work? (Use the bag or choose a multiple from 4 to 30)

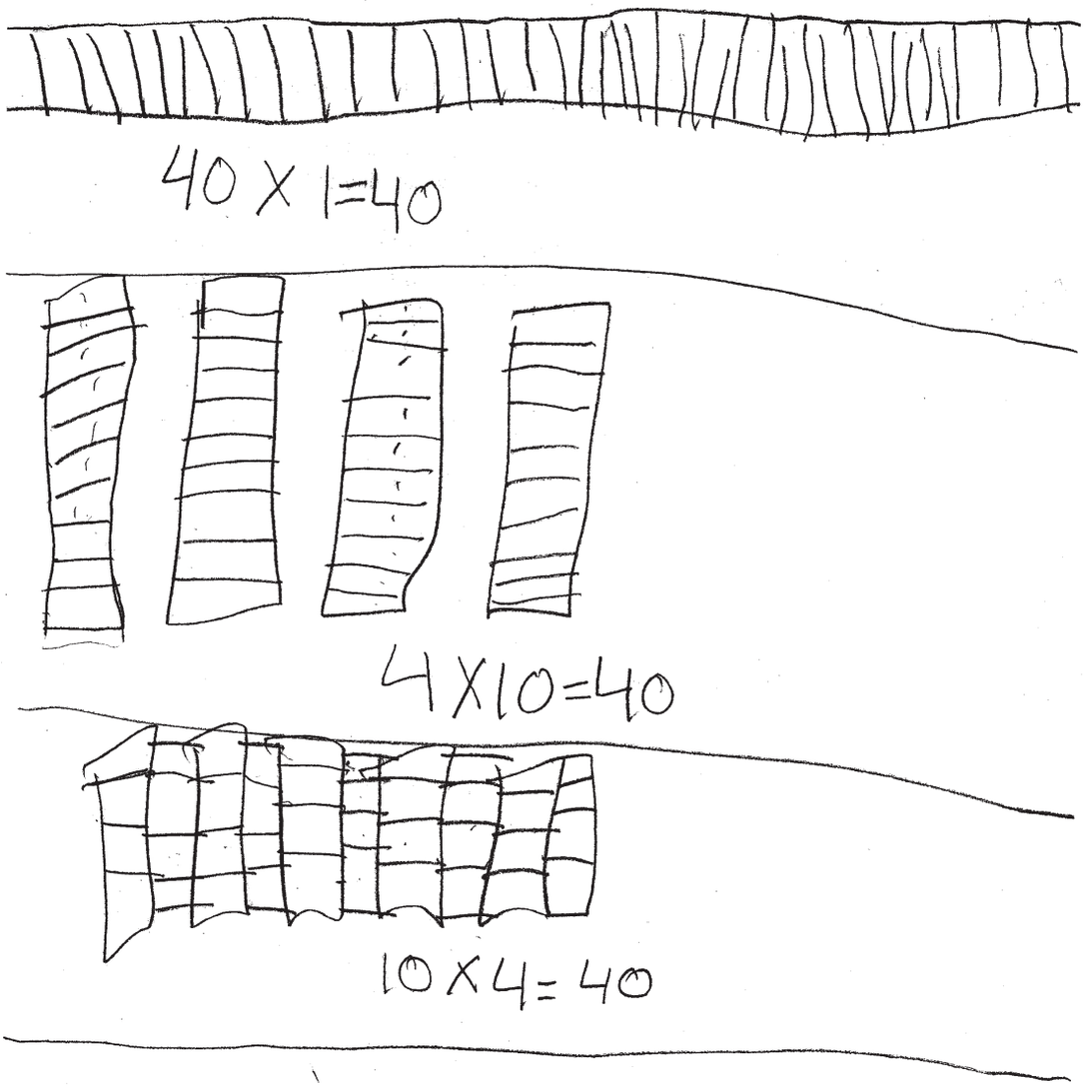


8 X 5 = 40



1 X 40 = 40

WORKSHOP 1 **STUDENT WORK SAMPLES**



WORKSHOP 1 HOMEWORK ASSIGNMENT

In preparation for the next Math for All Workshop, please complete the following assignments:

1. Conduct an observation of your focal child during a math lesson. Please follow the instructions below and use the Lesson Planning Chart to note the child's strengths and needs. After you have completed the observation, please answer the reflection questions.
2. Make sure to bring the completed Lesson Planning Chart and your answers to the reflection questions to the next workshop session. The facilitators may ask you to share your observations and will collect your charts and reflections so they can learn more about the children in your classrooms and your thinking about them. Feel free to submit the Demands of the Task and Observation Charts *as a group* if you worked on them together. However, please answer and submit the Reflection Questions *individually*.
3. Please read one of the following selections:
   - Chapters 2 ("Bringing the Science of Learning into the Classroom") and 3 ("Key Ingredients of Learning") from *Schools for All Kinds of Minds* (Barringer, Pohlman, & Robinson, 2010).
   - Chapters 2 ("The Ways of Learning") and 11 ("Getting a Mind Realigned") from *A Mind at a Time* (Levine, 2002).
4. Bring the textbooks and/or other curriculum materials you will be using between Workshop 2 and Workshop 3 to the next workshop session. You will need to use the books during the workshop to do some planning for a lesson that you will be teaching between the second and third workshop session.

PREPARING FOR AND CONDUCTING YOUR OBSERVATION OF A CHILD

1. Together with the members of your team, select a math lesson that you will teach over the next few weeks in which you can observe your focal child.
2. Read the description of the lesson and enact it with your colleagues (actually do the work of the lesson; don't just think about how it will be done). With your team, analyze the demands of the lesson, or a specific task within that lesson. Write down your conclusions in the second column of the Demands of the Task chart.
3. Conduct a 10- to 15-minute observation of your focal child in the lesson you selected. Take notes on the observation chart. Make sure to note both strengths and needs.
4. Answer the Reflection Questions.

WORKSHOP 1 **DEMANDS OF THE TASK CHART**

Your Name: _____ When Lesson Will Be Taught: _____

Name of Activity or Lesson Explored: _____

Learning Areas (based on Barringer et al., 2010; Levine, 2002; Pohlman, 2008)	What role do these learning areas play in the lesson?
Higher Thinking • using and forming concepts • solving problems • logical thinking • creative and critical thinking	
Language • understanding mathematical language • using language to communicate with others and to clarify one's ideas	
Spatial Ordering • interpreting relationships within and between spatial patterns • storing and recalling shapes, symbols, imagery, and appearances • organizing things in space (physical tools, workspace, information/data) • reasoning and conceptualizing with images	
Sequential Ordering • organizing information in sequence • following directions • managing time	

WORKSHOP 1 DEMANDS OF THE TASK CHART

Memory • short-term memory • active working memory • long-term memory	
Attention • controlling mental energy • maintaining focus • self-monitoring	
Psychosocial • using and understanding social language • collaboration • conflict resolution	
Motor Coordination • gross motor functions • fine motor functions • grapho-motor functions	

WORKSHOP 1 **OBSERVATION CHART**

Your Name: _____ Focal Student Pseudonym: _____

Name of Activity or Lesson Explored: _____

Learning Areas (based on Barringer et al., 2010; Levine, 2002; Pohlman, 2008)	How did the focal student respond to the various demands of the activity or lesson? Please note strengths and needs below
Higher Thinking • using and forming concepts • solving problems • logical thinking • creative and critical thinking	
Language • understanding mathematical language • using language to communicate with others and to clarify one's ideas	
Spatial Ordering • interpreting relationships within and between spatial patterns • storing and recalling shapes, symbols, imagery, and appearances • organizing things in space (physical tools, workspace, information/data) • reasoning and conceptualizing with images	
Sequential Ordering • organizing information in sequence • following directions • managing time	
Memory • short-term memory • active working memory • long-term memory	

WORKSHOP 1 OBSERVATION CHART

Attention • controlling mental energy • maintaining focus • self-monitoring	
Psychosocial • using and understanding social language • collaboration • conflict resolution	
Motor Coordination • gross motor functions • fine motor functions • grapho-motor functions	

WORKSHOP 1 **WORKSHEET 1G: LEARNING GOALS**

Your Name: _____ When Lesson Will Be Taught: _____

Focal Student Pseudonym: _____

Name of the Lesson: _____

Review the introductory pages for the unit of the Arranging Chairs lesson to help you answer the questions below. The unit overview is found on pages 17 (1.12) to 27 (1.22). The lesson guide is on pages 28 (1.23) to 35 (1.30).

1. What are the learning goals of the lesson?

2. In what ways do you think this lesson connects to what students have studied in math before (this school year and before)?

3. How do you think what students learn in this lesson will help them with the math they will learn in the future (this school year and beyond)?

WORKSHOP 1 REFLECTION QUESTIONS

Your Name: _____ Focal Student Pseudonym: _____

1. In what ways did your hands-on exploration and analysis of the lesson/math activity help you think about its demands? Did you learn anything new about the lesson/activity? Please explain.

2. How did observing a child enhance your understanding of how the child learns and his or her strengths and needs?

3. What things were hard about your assignment (conducting an observation of a child)?

4. What are the implications of what you learned about this child for your work with this child and other students in your class?