

CHAPTER 3. SPECIALIZED EXTENSIONS

We have by no means exhausted the possibilities of LGM with the examples presented thus far. As scientific hypotheses become more complex, the models used to represent those hypotheses will show a concomitant increase in complexity and flexibility. In this chapter, we present several interesting and useful extensions and specialized applications of traditional growth curve models that take advantage of both recent advancements in statistical theory and recent software improvements. These applications include growth mixture models, piecewise growth curve models, modeling change in latent variables, structured latent curve models, autoregressive latent trajectory models, and modeling change in categorical outcomes.

Growth Mixture Models

In most of the models described to this point, we assumed that a single latent trajectory would be sufficient to characterize the pattern of change in the population while allowing for random error in that trajectory. In Model 5, we noted that discrete classes (such as male and female) could follow different latent trajectories. That is, separate growth trajectories may be estimated simultaneously within each of several known groups, with or without cross-group constraints on key parameters. However, it need not be the case that the classification variable is observed; it is quite possible that latent (unobserved) classes could give rise to heterogeneous trajectories. If more than one such class exists, but only one trajectory is specified, significant bias likely will be introduced and the resulting trajectory may misrepresent all trajectory classes (Sterba, Prinstein, & Cox, 2007; von Eye & Bergman, 2003).

If it is reasonable to assume the existence of latent sources of heterogeneity in trajectories, then the researcher may wish to employ *latent growth mixture modeling* (LGMM). In a growth curve mixture model, the population is assumed to consist of a mixture of K homogeneous subgroups, each with its own distinct developmental trajectory. There are two popular approaches to fitting growth curve mixture models (B. Muthén, 2001; Nagin, 1999; Nagin & Tremblay, 2001). Both versions involve regressing the latent intercept and slope factors onto a latent classification variable, and both versions permit the form of the trajectory to differ across classes (i.e., the trajectory could be linear in one class and quadratic in another). The primary difference between the two approaches is that Muthén's

(2001) permits variability in trajectories within classes, whereas Nagin's (1999) requires trajectory variability to lie at the between-class level. As a consequence, Nagin's (1999) method usually results in concluding that more latent classes exist than does Muthén's (2001).

One potential disadvantage of LGMM is that growth mixture models may lead researchers to believe that multiple homogeneous subgroups exist, when in fact only one group exists in which the data are distributed nonnormally or follow a nonlinear trend (Bauer & Curran, 2003a). In other words, the groups identified as a result of growth mixture modeling may not represent true groups, but rather components of a mixture distribution of trajectories that together approximate a single nonnormal distribution. Because the existence of heterogeneous subpopulations is a basic assumption of LGMM, the method cannot *prove* that there are K classes, just as the implicit extraction of one class in traditional LGM does not *prove* there is only one population trajectory (Bauer & Curran, 2003a, 2003b, 2004). More generally, there is considerable evidence that a variety of assumption violations will produce artifactual latent classes (Bauer, 2005, 2007).

Growth mixture models are not easy to understand or to implement. Improper solutions are common, overextraction of classes is routine, and parameter estimation tends to be sensitive to starting values. In addition, model evaluation and model selection are not straightforward. Models specifying different numbers of classes may be compared using information criteria such as the Bayesian information criterion (BIC), but the application of information criteria to mixture models remains an active area of research. Several subjective decisions need to be made at various points in the process, and mixture modeling typically requires much larger sample sizes than standard applications. In addition, although assumption violations are always a potential hazard in latent variable analysis, LGMM is particularly vulnerable. Violation of distributional assumptions and misspecification of trajectory form can result in the extraction of multiple classes even in a homogeneous population. Nevertheless, there are exciting possibilities afforded by these methods. For example, LGMM can be used to test developmental theories involving equifinality, in which different initial conditions result in the same outcome, and multifinality, in which identical initial conditions lead to different outcomes (e.g., Cicchetti & Rogosch, 1996).

LGMM is an area of active study, and advances continue to be made. For example, Klein and Muthén (2006) describe an extension to LGMM that permits heterogeneity in growth to depend on initial status and time-invariant covariates. Their method results in more accurate prediction intervals than standard LGM without being as highly parameterized as LGMM. The method is not yet implemented in widely available software.

At present, growth mixture modeling can be accomplished with MECOSA for GAUSS (Arminger, Wittenberg, & Schepers, 1996), Mplus (L. K. Muthén & Muthén, 1998–2006), Mx (Neale et al., 2003), and SAS PROC TRAJ (Jones, Nagin, & Roeder, 2001). Mplus is currently the most flexible of these applications. Interested researchers are referred to Bauer (2005, 2007), Bauer and Curran (2003a, 2003b, 2004), T. E. Duncan et al. (2006), Li, Duncan, Duncan, and Acock (2001), and M. Wang and Bodner (2007).

Piecewise Growth

Suppose theory or prior research suggests that growth should proceed at a different—but still linear—rate during the middle school years than during the elementary school years (due to differences in school funding, for example). Both phases of growth may be modeled within a single LGM using a *piecewise growth model* (T. E. Duncan et al., 2006; Sayer & Willett, 1998) or *discontinuity design* (Hancock & Lawrence, 2006). Piecewise growth models are specified by including two or more linear slope factors in one model. For example, if we had continued to collect data for Grades 7, 8, and 9 and hypothesized that growth would decelerate in the middle school years, the loading matrix Λ_y may look like Equation 3.1:

$$\Lambda_y = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 4 & 0 \\ 1 & 5 & 0 \\ 1 & 5 & 1 \\ 1 & 5 & 2 \\ 1 & 5 & 3 \end{bmatrix}. \quad (3.1)$$

In Equation 3.1, the first column represents the intercept, the second column represents linear growth up to the sixth grade (the fifth occasion of measurement), and the mean of the first linear slope factor will represent the rate of linear growth characterizing the elementary school years. The third column of Λ_y represents linear growth during the middle school years, treating the last year of elementary school as a second origin for the time scale. The mean of the second linear slope factor will reflect the middle school rate of growth. An alternative (but statistically equivalent) specification might be to use the Λ_y matrix in Equation 3.2 (Hancock & Lawrence, 2006). In Equation 3.2, the first slope factor represents linear growth across Grades 1 to 9, and the second slope factor represents any *additional* linear

change, beginning in Grade 7, above and beyond that captured by the second linear slope:

$$\mathbf{A}_y = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 4 & 0 \\ 1 & 5 & 0 \\ 1 & 6 & 1 \\ 1 & 7 & 2 \\ 1 & 8 & 3 \end{bmatrix}. \quad (3.2)$$

Recent methodological work suggests exciting possibilities for modeling change that occurs in discrete segments. Cudeck and Klebe (2002) describe *multiphase models* for longitudinal data within the multilevel modeling framework. Multiphase regression models can be used to model change in multiple growth periods, each characterized by different functional forms (e.g., a downward linear slope followed immediately by an upward linear slope). The point of transition from one period (or *regime*) to another is known as a *change point* or *knot*. In theory, these change points may be modeled as aspects of change in their own right—as fixed parameters, estimated parameters, or random coefficients. For example, Cudeck and Klebe model the sharp quadratic growth, and subsequent gradual linear decline, in nonverbal intelligence across the life span. They estimated the age of transition from the first phase to the second phase as a random coefficient with a mean of 18.5 years and a Level 2 variance of 9.25.

Specifying multiphase models in the LGM framework is not always possible, but the ability to examine model fit, use aspects of change as predictor variables, and assess multiphase change in latent variables makes it a worthwhile topic to explore. Multiphase models with known change points may be examined by using partitioned \mathbf{A}_y matrices, with each partition containing the basis curves for the corresponding segment of the trajectory. For example, the \mathbf{A}_y matrix in Equation 3.3 represents a model for equally spaced occasions with an intercept and linear slope in the first segment (with means α_1 and α_2), a different intercept and linear slope in the second segment (with means α_3 and α_4), and a change point where *time* = 2.3. Alternatively, an unknown change point ω may be estimated as a function of model parameters when the \mathbf{A}_y matrix is specified as in Equation 3.4, in which the time metric picks up in the fourth column where it left off in the second. Assuming that the segments ($\alpha_1 + \alpha_2 \text{time}$) and ($\alpha_3 + \alpha_4 \text{time}$) are continuous at *time* = ω , the values implied for *y* by both segments of the

multiphase trajectory must be equal at ω . One of the parameters (say, α_3) is therefore redundant and can be eliminated by constraining it to equal a function of the other parameters, $\alpha_3 = \alpha_1 + (\alpha_2 - \alpha_4)\omega$, and estimating ω as a parameter. Similar algebra may be used in growth models with more complicated growth functions in multiple segments. There is currently no straightforward way to model change points as random coefficients in the LGM framework, although this can be done with multilevel modeling (see Cudeck & Klebe, 2002).

$$\Lambda_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0.7 \\ 0 & 0 & 1 & 1.7 \\ 0 & 0 & 1 & 2.7 \end{bmatrix}, \quad (3.3)$$

$$\Lambda_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}. \quad (3.4)$$

Modeling Change in Latent Variables With Multiple Indicators

None of the models presented earlier takes full advantage of one of the most basic and useful features of SEM—the ability to model relationships among latent variables with multiple measured indicators. Up to this point, we have said nothing about reliability or measurement error, but in fact, it is rare to encounter variables with near-perfect reliability in the social sciences. Routine applications of SEM explicitly model unreliability by partitioning observed variability into common variance (variance shared by a group of measured variables) and unique variance (a combination of measurement error and reliable variance specific to a variable). In most SEM applications, the diagonal elements of Θ_ϵ represent unique variance, but in LGM, the diagonal elements of Θ_ϵ represent a combination of measurement error and departure from the mean trend at each measurement occasion. To separate common from unique variance, repeated measures may be represented by latent variables and multiple time-specific indicators may be directly incorporated in the model.

These models are sometimes known as *curve-of-factors models* (S. C. Duncan & Duncan, 1996; McArdle, 1988), *latent variable longitudinal curve models* (Tisak & Meredith, 1990), or *second-order latent growth models* (Hancock, Kuo, & Lawrence, 2001; Hancock & Lawrence, 2006; Sayer & Cumsille, 2001); the repeated-measure latent variables are termed *first-order factors* and the growth factors (i.e., intercept and slope) are termed *second-order factors*. An example is presented in Figure 3.1.

Several advantages accrue by using latent repeated measures when multiple indicators are available. First, the second-order LGM explicitly recognizes the presence of measurement error in the repeated measures and models growth using latent variables adjusted for the presence of this error. Second, second-order growth curve models allow the separation of disturbance variation due to departure from the mean trend (temporal instability, reflected by $\psi_{33}-\psi_{66}$ in Figure 3.1) and unique variation due to measurement error (unreliability, reflected by Θ_{ϵ} in Figure 3.1). Third, second-order growth curve models permit tests of longitudinal factorial invariance, or *stationarity* (Sayer & Cumsille, 2001; Tisak & Meredith, 1990). It is extremely important that the latent variable of interest retain its meaning throughout the period of measurement (Willett, 1989). For this assumption to be supported, the factor structure should be invariant over repeated occasions (Chan, 1998; Meredith & Horn, 2001). That is, at the very least, factor loadings for similar items should be the same over repeated measures. Although it is beyond the scope of this book to delve into issues surrounding longitudinal factorial invariance, we cannot overemphasize its importance for studies of growth over time. For more details on specifying second-order growth curve models, consult Chan (1998), Hancock et al. (2001), and Sayer and Cumsille (2001).

Structured Latent Curves

The polynomial growth functions considered earlier are characterized by a property known as *dynamic consistency*. Loosely, dynamic consistency refers to the property that the “average of the curves” follows the same functional form as the “curve of the averages” (Singer & Willett, 2003). This property holds for linear growth, quadratic growth, and indeed any growth function that consists of a weighted linear composite of functions of time. A convenient consequence of dynamic consistency is that the first derivatives of the growth function with respect to growth parameters (if expressed in traditional polynomial form) are simply numbers, which can be coded directly into the Λ_y matrix using virtually any SEM software application. But growth functions in LGM need not be limited to the polynomial curves suggested by Meredith and Tisak (1990).

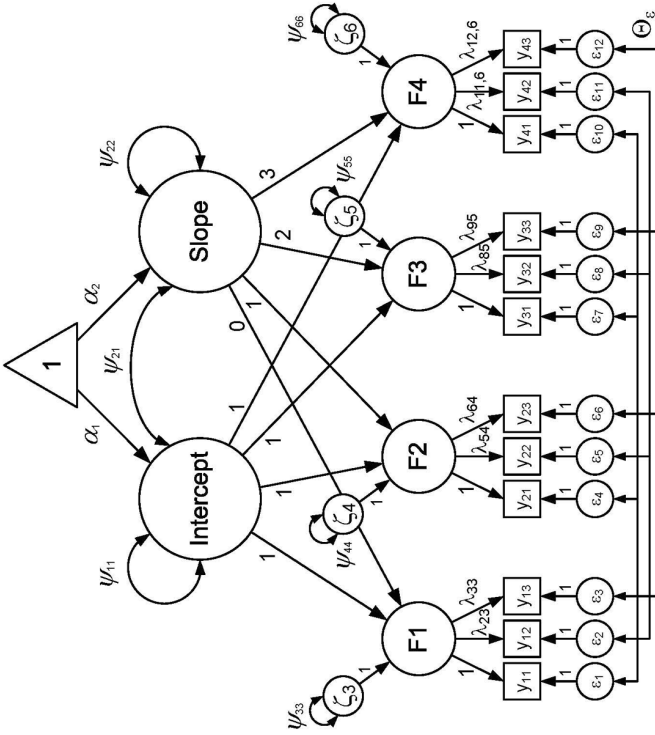


Figure 3.1 A Second-Order Growth Curve Model, Representing Linear Change in Four Repeated Measures of a Latent Variable With Three Measured Indicators.

NOTE: Included in the model, but not pictured, are paths (τ_i parameters) from the Triangle Constant 1 to the measured indicators not serving as scaling indicators (see Hancock et al., 2001). We chose to allow the unique factors of similar indicators to covary across repeated measures to acknowledge that covariances among indicators measured at different occasions are not expected to be completely explained by mutual reliance on growth factors.

Browne and du Toit (1991) and Browne (1993) propose and illustrate a *structured latent curve* (SLC) approach to modeling nonlinear growth functions not characterized by dynamic consistency. In SLC models, the loadings in Λ_y may assume values consistent with any hypothesized growth function $f(t, \boldsymbol{\theta})$ (a function of time, t , and growth parameters, $\boldsymbol{\theta}$), referred to as the *target function* (Blozis, 2004). The function $f(t, \boldsymbol{\theta})$ is assumed to be smooth and differentiable with respect to elements of $\boldsymbol{\theta}$. The SLC method of specifying growth models is a direct extension of Rao's (1958) EFA method of obtaining parameters for growth functions. The elements of Λ_y are not specified as fixed values. Rather, they are estimated, but are required to conform to basis curves consistent with $f(t, \boldsymbol{\theta})$. The polynomial curves considered in previous examples are special cases of this more general framework.

To understand the SLC framework, it is helpful to recognize that the loadings specified in the Λ_y matrix for polynomial growth curve models correspond to the first partial derivatives of the hypothesized growth function with respect to each growth parameter. For example, for the quadratic growth curve specified in Model 10, the target function is

$$\hat{y}_{it} = \theta_1 + \theta_2 t_{it} + \theta_3 t_{it}^2; \quad t = \{0, 2, 3, 4, 5\}. \quad (3.5)$$

The first derivatives of this function with respect to each growth parameter are, respectively,

$$\frac{\partial \hat{y}_{it}}{\partial \theta_1} = 1, \quad (3.6)$$

$$\frac{\partial \hat{y}_{it}}{\partial \theta_2} = t, \quad (3.7)$$

and

$$\frac{\partial \hat{y}_{it}}{\partial \theta_3} = t^2, \quad (3.8)$$

which are known quantities. Thus,

$$\Lambda_y = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}. \quad (3.9)$$

More complex—but dynamically inconsistent—growth functions, such as exponential, Gompertz, and logistic curves, may be specified in a similar manner. For growth functions that are not dynamically consistent, the growth parameters in $\boldsymbol{\theta}$ may not reduce to simple functions of t , and may instead require more complicated specifications of $\boldsymbol{\Lambda}_y$. For example, consider exponential growth. If an exponential process were appropriate for the mother–child closeness data (it is not, but let us pretend), the target function would be

$$\hat{y}_{it} = \theta_1 - (\theta_1 - \theta_2)e^{(1-t_{it})\theta_3}; \quad t = \{1, 3, 4, 5, 6\}. \quad (3.10)$$

The first derivatives of this function with respect to each growth parameter are, respectively,

$$\frac{\partial \hat{y}_{it}}{\partial \theta_1} = 1 - e^{\theta_3(1-t_{it})}, \quad (3.11)$$

$$\frac{\partial \hat{y}_{it}}{\partial \theta_2} = e^{\theta_3(1-t_{it})}, \quad (3.12)$$

and

$$\frac{\partial \hat{y}_{it}}{\partial \theta_3} = (\theta_1 - \theta_2)(t_{it} - 1)e^{\theta_3(1-t_{it})}. \quad (3.13)$$

Thus, if we let the initial occasion $t_{it} = 1$,

$$\boldsymbol{\Lambda}_y = \begin{bmatrix} 0 & 1 & 0 \\ 1 - e^{-2\theta_3} & e^{-2\theta_3} & 2(\theta_1 - \theta_2)e^{-2\theta_3} \\ 1 - e^{-3\theta_3} & e^{-3\theta_3} & 3(\theta_1 - \theta_2)e^{-3\theta_3} \\ 1 - e^{-4\theta_3} & e^{-4\theta_3} & 4(\theta_1 - \theta_2)e^{-4\theta_3} \\ 1 - e^{-5\theta_3} & e^{-5\theta_3} & 5(\theta_1 - \theta_2)e^{-5\theta_3} \end{bmatrix}. \quad (3.14)$$

For polynomial curves, θ_1 , θ_2 , and θ_3 are estimated as means of three basis curve factors. In the exponential SLC, however, θ_1 and θ_2 are estimated as means but θ_3 is estimated using LISREL's additional parameter feature. Detailed treatments of this kind of model are presented in Browne (1993) and Blozis (2004). Blozis (2006) extends this method to model nonlinear trends in latent variables measured with multiple indicators, and Blozis (2007) extends the method to explore multivariate nonlinear change. Specialized software (e.g., AUFIT; du Toit & Browne, 1992) is required to estimate most SLC models, but LISREL can be used to estimate many such models using complex equality constraints.

Autoregressive Latent Trajectory Models

Although LGM is a useful and flexible approach for investigating change over time, there are alternative SEM-based strategies. One alternative model is the *autoregressive* or *Markov simplex* model proposed by Guttman (1954), in which each repeated measure is regarded as a function of the preceding measure and a time-specific disturbance term. Unlike the traditional LGM, the simplex model is not concerned with trends in the mean structure over time, but rather with explaining variance at each wave of measurement using the previous wave. Curran and Bollen (2001) and Bollen and Curran (2004, 2006) propose the *autoregressive latent trajectory* (ALT) model that combines features of both the LGM and simplex models. One example of an ALT model is depicted in Figure 3.2. The ALT model differs from the standard LGM model in two main respects. First, in the pictured parameterization of the ALT model, there is no disturbance term associated with the first repeated measure. Second, like the autoregressive or simplex model, directional paths (the ρ parameters in Figure 3.2) are specified to link adjacent repeated measures. It is common practice in specifying both the simplex model and ALT models to constrain the $\rho_{t,t-1}$ parameters to equality, although this constraint is by no means necessary. The ALT model is related to one of the TVC models presented earlier (Model 9; see Table 2.12), in which the value of the outcome variable at time $t - 1$ is used as a predictor of the outcome at time t . Applications of univariate and multivariate ALT models can be found in Rodebaugh, Curran, and Chambless (2002) and Hussong, Hicks, Levy, and Curran (2001). A closely related model is the *latent difference score* model (McArdle, 2001; McArdle & Hamagami, 2001).

Categorical and Ordinal Outcomes

Our discussion so far has assumed that the repeated-measures possess interval or ratio characteristics. However, many variables in the social sciences are more properly treated as ordinal. It is difficult to conclude, for example, that dichotomous items or Likert-type scales with only three or four choices have interval properties (although with a sufficient number of choices, many rating scales may be treated as if they were continuous). When the repeated observed variable is ordinal, the data consist not of means and covariances but rather a potentially large contingency table, with each cell containing the number of respondents matching a particular response pattern. If the ordinal data can be assumed to reflect an underlying normally distributed latent variable, a multistage estimation procedure may be used

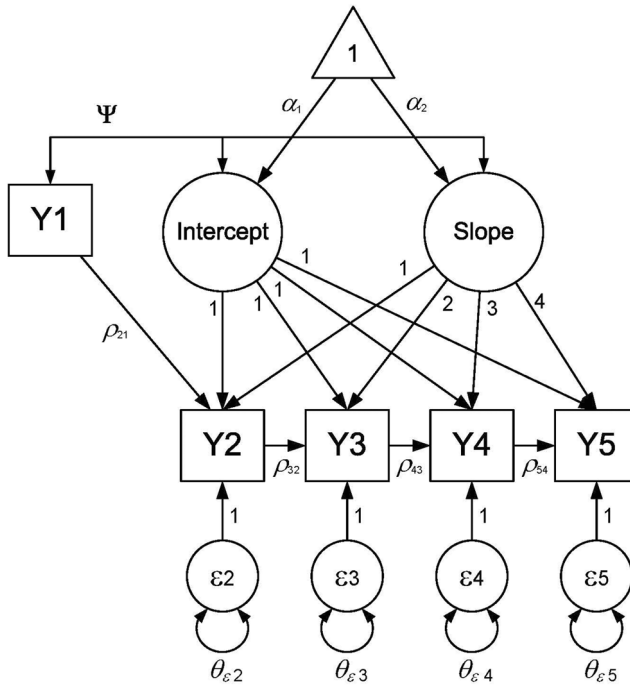


Figure 3.2 An Autoregressive Latent Trajectory Model for Five Repeated Measures.

to model growth (Mehta, Neale, & Flay, 2004). This procedure assumes that unobserved thresholds on the latent distribution at each occasion determine the multivariate probabilities associated with all response patterns. In the first stage, a link function is specified to model the multivariate ordinal response probabilities and a likelihood function is estimated for each case. In the second stage, a common measurement scale is established for the unobserved latent response variates, which are assumed to be normally distributed. Response thresholds are assumed to remain invariant over time. In the third stage, a growth model is fit to the scaled latent response variables. Currently, only a few software programs are capable of estimating such models, for example, Mplus and Mx. If some data are missing, Mx is the only option currently available. Jöreskog (2002) describes a similar procedure using LISREL, which involves fitting the model to a polychoric covariance matrix and means estimated from raw ordinal data.

The application of LGM to categorical data is receiving much attention in the methodological literature. For example, Liu and Powers (2007) recently described a method to model zero-inflated count data within the LGM framework. Many interesting advances are expected to occur in the foreseeable future. For more detail on methods for estimating growth models using binary or ordinal data, see Jöreskog (2002), Mehta et al. (2004), and B. Muthén and Asparouhov (2002).

Modeling Causal Effects Among Aspects of Change

To this point, intercept and slope (co)variances have been “unstructured” in the sense that they have been permitted to freely covary. We may instead elect to estimate directional effects among aspects of change; that is, we can model aspects of change as functions of other aspects of change. For example, if the time metric is centered at the final occasion of measurement, it may be of interest to model “endpoint” as a function of rate of change by regressing the intercept factor onto the slope factor. Alternatively, we could elect to center time at the initial occasion and model slopes as a linear function of intercepts. Caution is warranted here, however. Causes must logically precede effects, so it would be causally inconsistent to regress slopes on intercepts unless the time origin occurs at or before the initial measurement.

Muthén and Curran (1997) creatively capitalize on this feature of SEM to model treatment effects in situations where participants are randomly assigned to (at least) two groups and repeatedly measured on some outcome of interest, such as in intervention studies. They suggest fitting the same growth curve to both groups, constraining the linear growth parameters to equality across groups (see Figure 3.3). A second slope factor (Tx Slope) is added to the model for the experimental group. The control group thus provides a baseline trajectory against which the experimental group's trajectory may be compared. Any additional change observed beyond that associated with the first slope factor is due to the treatment effect. An important aspect of their model is that the additional treatment slope factor may be regressed on the intercept factor (initial status), allowing the examination of an intercept \times treatment interaction. For example, it may be the case that mother-child pairs with relatively low initial closeness may benefit more over time from an intervention targeting the prevention of externalizing behaviors. In Muthén and Curran's model, the intercept factor would influence a third latent variable (intervention), which in turn would affect the measured variable across time, but only in the experimental group.

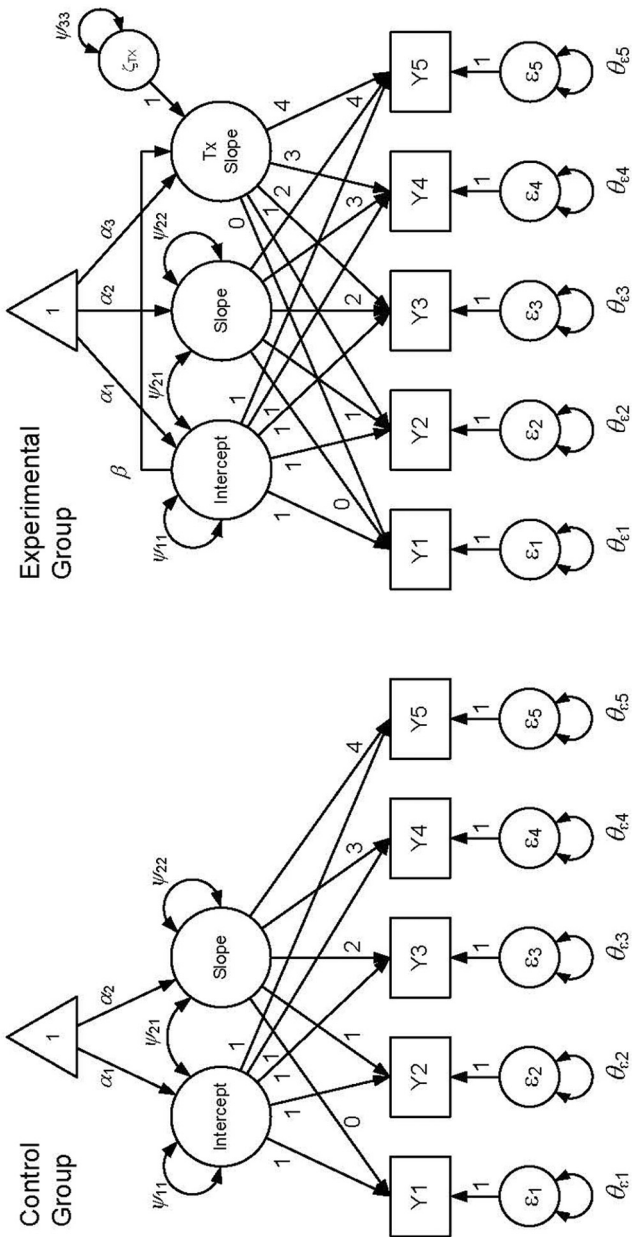


Figure 3.3 Muthén and Curran's (1997) Two-Group Growth Model for Intervention Studies.

Summary

In this chapter, we have described several specialized extensions of LGM that demonstrate its generality and flexibility as an analytic tool.

It is possible to combine features of the models discussed here and to expand these models in other ways, depending on the researcher's requirements. For example, it is feasible to explore growth mixture models applied to cohort-sequential data and to examine the effects of TVCs or exogenous predictors in parallel process ALT models (Bollen & Curran, 2004; Curran & Willoughby, 2003). Simons-Morton, Chen, Abrams, and Haynie (2004) model three parallel processes (adolescent smoking, friend smoking, and parental involvement), including time-invariant predictors and specifying effects of intercepts on slopes both within and between processes. Sayer and Willett (1998) combine piecewise and parallel process growth models, fitting them simultaneously in two groups. McArdle (1989) describes a multiple-groups parallel process model. Finally, we did not present an example of this kind of model, but it is straightforward to treat intercepts and slopes as predictors of outcome variables. For example, it might be important to test the hypothesis that rate of acquisition of a skill predicts individual differences in ability years later. Muthén and Curran (1997) present some models in which intercept and slope factors are used to predict a distal outcome or to predict intercept and slope factors associated with a different repeated-measures variable.

The LGM framework permits specification and testing of several kinds of interaction or moderation hypotheses. For example, researchers can test hypotheses about the interaction among two or more exogenous predictors of slope (Curran et al., 2004; Li, Duncan, & Acock, 2000; Preacher et al., 2006), between initial status and time in determining an outcome (Muthén & Curran, 1997), or between time and a TVC (see discussion of Model 9 in Chapter 2). LGM can be extended to three (or more) levels of hierarchically organized data by employing multilevel SEM (Mehta & Neale, 2005; B. Muthén, 1997). At the other end of the spectrum, it is also possible to fit simple ANOVA, MANOVA, and simplex models in LGM as special cases (Meredith & Tisak, 1990).