WHAT YOUR COLLEAGUES ARE SAYING . . .

"The wait is over! Katie Lewis, Amy Anderson, and Dylan Lynn have written the first book on dyscalculia that is strengths-focused, research-based, and accessible to all who work with students with dyscalculia. Lewis and colleagues provide the most accurate, up-to-date information on dyscalculia in a practitioner-friendly format. I learned so much from the description of inaccessible mathematical notation, which has impacted my teaching. In addition, they provide structure to help educators use clinical interviews to understand what students know, what they are working on, and (crucially!) what students understand about their own learning processes in math. I can't wait to use this text in many different contexts to support students and teachers!"

—**Rachel Lambert**, Associate Professor, Gevirtz Graduate School of Education, UC Santa Barbara; Author, *Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K–8*, Santa Barbara, CA

"This book provides a pathway through which to explore and understand anti-deficit perspectives about disability, specifically, and mathematics learning, broadly. We are reminded that disabled students' knowing and ways of being mathematical are interesting, legitimate, and necessary. Grounded in theory, research, and disabled students' thinking, this book provides practical approaches that leverage the best of mathematics education and special education."

—**Erica N. Mason**, Assistant Professor, University of Illinois Urbana–Champaign, Champaign, IL

"What if dyscalculia is not something to fix, but a mathematical world to understand? *Exploring Dyscalculia* approaches this terrain with care, rigor, and hope. Challenging deficit myths, it offers clear, practical guidance for identifying inaccessibility, honoring students' strengths, and navigating barriers through rich examples that bring their mathematical thinking to life."

—**Cathery Yeh**, Associate Professor of Mathematics Education, The University of Austin, Austin, TX

"Finding and addressing the causes of your students' mathematical difficulties can be a real challenge. This book is a big help! Loaded with examples, the book explains what dyscalculia is. It shows how to identify some of the reasons your students find math so difficult, and ways you can provide the kinds of support that helps them learn."

—**Alan H. Schoenfeld**, Distinguished Professor of Education and Mathematics, UC Berkeley, Berkeley, CA

"Finally! A book that unpacks both the patterns and uniqueness of dyscalculia in students. And the authors do it all from an anti-deficit framing. This is the guidebook educators have needed for decades. Perfect for both math and special education teachers. I devoured every story, strategy, and suggestion!"

—**Liesl McConchie**, Owner, Math With the Brain in Mind; Author, *Building a Positive Math Identity: A Brain Science Approach*, San Diego, CA

"This text bridges mathematics and special education research and practice in ways that feel both inspiring and immediately usable. The vignettes, thoughtful instructional considerations, and ready-to-use assessment tools make it invaluable for teacher candidates and transformative for in-service professional learning. A must-read for anyone committed to asset-based, evidence-based mathematics instruction."

—**Sararose Lynch**, Associate Professor, Slippery Rock University, Slippery Rock, PA

"This groundbreaking book fills my heart with joy! Katherine Lewis, Amy Anderson, and Dylan Lynn bring a wealth of knowledge guiding teachers to understand dyscalculia through detailed conversations with students. My copy of this book is going to be well worn as I return to it again and again. With deep respect for students with learning disabilities, you will learn how to tackle what makes mathematics inaccessible to students with disabilities and instead help them thrive."

—**Elham Kazemi**, Professor, Mathematics Education, University of Washington, Seattle, WA

"Lewis, Anderson, and Lynn present some compelling research about the unique ways students with dyscalculia engage with math (specifically quantities, symbols, and notation) that encourages a much deeper understanding of our students than previously possible."

—**James Manzano**, Math Learning Specialist, Community Roots Charter School, Ridgewood, NY

"This book fills a necessary void in applicable research for teachers of students who struggle with math. While information is readily available to support special educators and teachers in dyslexia, dyscalculia is often misunderstood. In order to provide effective instruction and implement best practices, educators must understand the inaccessibilities that students with dyscalculia face. This book provides both general educators and special educators with research-based best practices for understanding, assessing, and supporting students with dyscalculia."

—**Emily Dwivedi**, Special Educator and Math Resource Teacher, Baltimore County Public Schools, Baltimore, MD

Exploring Dyscalculia

Learning From and Supporting Students

Katherine E. Lewis

Amy B. Anderson

Dylan M. Lynn

CORWIN
Mathematics

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United Kingdom

Sage Publications India Pvt. Ltd.
10th Floor, Emaar Capital Tower 2
MG Road, Sikanderpur
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China Square Central
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Vice President, Content and Product: Monica Eckman
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Typesetter: C&M Digitals (P) Ltd.
Proofreader: Scott Oney
Indexer: Integra
Cover Designer: Gail Buschman

Printed in the United States of America

Library of Congress Cataloging-in-Publication Data

Names: Lewis, Katherine, 1977- author | Anderson, Amy (Educational therapist) author | Lynn, Dylan author

Title: Exploring dyscalculia : learning from and supporting students / Katherine E. Lewis, Amy B. Anderson, Dylan M. Lynn.

Description: Thousand Oaks, California : Corwin, [2027] | Includes bibliographical references and index.

Identifiers: LCCN 2026021345 | ISBN 9798348808372 paperback | ISBN 9798348863593 epub | ISBN 9798348863609 epub | ISBN 9798348863616 pdf

Subjects: LCSH: Acalculia | Acalculia in children | Mathematics—Study and teaching | Learning disabled children—Education

Classification: LCC RJ496.A25 L48 2027
LC record available at https://lccn.loc.gov/2026021345

This book is printed on acid-free paper.

26 27 28 29 30 10 9 8 7 6 5 4 3 2 1

CONTENTS

online resources

Visit the companion website at
http://klewismath.com/dyscalculia/
for downloadable resources.

Note From the Publisher: The authors have provided web content throughout the book that is available to you through a QR (quick response) code. To read this QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Web content may also be accessed at qrs.ly/xdh9e48

PREFACE

We wrote this book because far too many students, parents, caregivers, and teachers have turned to us desperate for understanding, guidance, and answers. Students with dyscalculia are frequently told they are lazy and blamed for their lack of progress in mathematics. Caregivers, who see the depth of their child's struggles, are often frantic to find resources to help their child succeed. Teachers are often unsure of how to support those students who just never seem to "get it." There is a general lack of understanding of dyscalculia, and many recommendations rely upon a problematic deficit framing of students. We offer an alternative, grounded in an anti-deficit perspective and drawing from our collective lived and professional experience.

We are educators and researchers who are all passionate about supporting students with dyscalculia. Katie Lewis, an associate professor at the University of Washington, is a researcher and teacher educator who studies disability in mathematics, focusing specifically on dyscalculia. Amy Anderson, an experienced teacher and educational therapist, specializes in online one-on-one instruction for students with disabilities. Dylan Lynn, a former statistician who has dyscalculia, consults with families and students on educational issues related to dyscalculia. We each come to this work with personal and professional experiences with disability.

This book is a curated collection of knowledge about dyscalculia. In it, we draw on mathematics education research and work from scholars and educators who take up anti-deficit approaches to studying dyscalculia. We aim to capture what dyscalculia looks like as students are in the process of learning and problem-solving. Many of the vignettes and examples we share throughout the book come from individual one-on-one instruction with students. These detailed examples enable us to see patterns in students' thinking that would be invisible in small-group or whole-class contexts and are essential to understanding the inaccessibility students with dyscalculia face. We also use detailed examples to illustrate how instructional practices that center student access and sense-making can empower students to engage in grade-level mathematics and navigate barriers they encounter. While we focus heavily on one-on-one instructional examples, we include assessment approaches, instructional practices, and resources for teachers and families to support students with dyscalculia in the classroom and at home.

If you are reading this book, it is likely that you, too, have encountered a student who has unusual difficulties with math. You may want to understand why. You may be eager to learn approaches to help support them. We hope the examples in this book will enable you to listen for the same kinds of difficulties in your own students and provide you with instructional approaches you can try. Supporting students with dyscalculia is an active and ongoing process of understanding their difficulties and providing support. We welcome you into this inquiry and problem-solving space.

ABOUT LANGUAGE CHOICES

In this book we take an anti-deficit perspective on disability. What this means is that, like many Disability Studies scholars and Disability Justice activists, we understand disability to result from natural and *desirable* biological human variation.

Disability is not a bad word. It is not a tragedy. It is not something to be cured. Disability is part of the natural variation that we have in our human species. We do not think of these biological differences as deficits within the individual; instead, we understand that individuals with disabilities are disabled by systems, structures, and societal expectations that are not accessible and welcoming. For example, a Deaf person is not disabled by their biological difference in hearing; instead, they are disabled by contexts that rely exclusively upon spoken communication. It is important to name these differences as disabilities (rather than a patronizing euphemism like differently abled), because individuals with disabilities experience oppression as they are forced to navigate inaccessible systems and contexts, and it is important to acknowledge and honor that.

An anti-deficit approach provides us with greater insight into our students and opens up more possibilities to support them. When we understand cognitive variability as natural and desirable, we realize the students' difficulties result because aspects of the mathematics itself are inaccessible. This shifts our focus away from misguided attempts to "fix" the student and toward modifying our instruction to create access. When we honor how the student is actively making sense of the mathematics, we can creatively work with them to determine what supports would be helpful. We believe that deficit-based beliefs about students with dyscalculia are harmful. Throughout the book we identify common deficit-based myths about dyscalculia, debunk these myths, and describe how you can counter them within the classroom.

The students with dyscalculia that are discussed in this book come from a range of racial, cultural, and socioeconomic backgrounds. Currently, there is not sufficient research about how identity differences may play out specifically for dyscalculia. However, there is considerable evidence from mathematics education that mathematics classrooms are racialized spaces, and that ableism and racism intersect. Discussion of the relationship of race, culture, language, and disability in mathematics is beyond the scope of this book. For readers interested in these topics, we highly recommend the work of Rachel Lambert, Paulo Tan, and Cathery Yeh.

We have written this book primarily for teachers. We draw upon data collected with students of all different ages; however, much of the content focuses on elementary mathematics. It is essential to understand how dyscalculia impacts students in elementary mathematics topics because high school and college students with dyscalculia may continue to have difficulties with elementary number concepts and notation. Because special education and mathematics teachers have different disciplinary knowledge, throughout the book, we will define both mathematical terms and special education terminology.

OVERVIEW OF THE BOOK

This book does not contain quick fixes, magic bullets, or "cures." Your students with dyscalculia will always have dyscalculia. Their brains will always work in their own brilliant, incredible, and unique ways. This does not mean that they are destined for failure. Just like dyslexic students learn to read and write (and some of us even go on to write books), students with dyscalculia can learn to create access for themselves and thrive in mathematics in school and in their careers.

In this book, we invite you to join us in exploring the terrain of dyscalculia. Students with dyscalculia will need to learn to navigate around the inaccessibilities they encounter in mathematics. As a teacher, you can help your students find their bearings. Their path through the inaccessibilities will not be straight or easy. In this book, we will also provide you with a map as you help them chart their course. We will tell you what things to keep an eye out for and what approaches have helped students circumnavigate the inaccessibilities. Supporting students with dyscalculia involves cycles of inquiry and experimentation. It also requires that we move from being the teaching expert and be humble enough to learn from our students about their experiences. Although this exploration is hard work for both the teacher and the student, it is incredibly rewarding. Helping a student with dyscalculia access mathematics and find joy in their own mathematical sense-making is an incredibly fulfilling experience.

To help you and your students explore and navigate dyscalculia, we have divided this book into two parts. Part I (Chapters 1–3) provides an overview of the terrain and describes the kinds of landmarks you may see along the way. Part II (Chapters 4–6) supports readers to dive into the cycle of inquiry and begin to chart and navigate inaccessibilities with your student. In each chapter, we present research-based examples from students with dyscalculia and then practical approaches about how you can embark on your own exploration with your students tomorrow.

Chapter 1 summarizes what is known about dyscalculia and illustrates the power of listening and learning from students. We provide a general overview of research on dyscalculia, but interpret the findings through an anti-deficit lens. Through examples of students engaged in learning and doing mathematics, we demonstrate how listening to our students can provide insight into the kinds of unique difficulties that students with dyscalculia experience. When we recognize these inaccessibilities, we can design instructional approaches that support the student in navigating around them.

In Chapter 2 and Chapter 3, we present the primary kinds of inaccessibilities that emerge for students with dyscalculia: inaccessible *quantities* and inaccessible *notation*. First, in Chapter 2, we dive into examples from students to explore how numbers and quantities are often inaccessible for students with dyscalculia. Students with dyscalculia tend to understand quantities as enacted processes rather than stable amounts. This results in difficulties perceiving, representing, manipulating, and remembering quantities and numbers. Then, in Chapter 3, we examine how symbolic notation is also often inaccessible for students with dyscalculia. We provide examples of how students with dyscalculia have difficulty reading, writing, remembering, or using mathematical notation. In both chapters, through detailed examples, we explore both how these inaccessibilities with quantity and notation can lead to persistent difficulties with mathematics and how students may compensate when they encounter an inaccessibility. Together, the three chapters in Part I aim to provide an understanding of what dyscalculia may look like when students attempt to learn and do mathematics. Being aware of what inaccessibilities look and sound like when they appear provides essential information about the uncharted terrain that you are entering with your student.

With this general knowledge of dyscalculia in mind, Part II focuses on how you can help support your student in charting their own path. This exploration involves cycles of inquiry and experimentation, to identify inaccessibilities and explore how students may navigate around them.

In Chapter 4, we describe a powerful assessment tool that can be used to understand the specific inaccessibilities your student experiences, the clinical interview. Clinical interviews allow the teacher to take on the role of an inquirer and identify students' strengths, difficulties, and inaccessibilities. A central idea in this form of assessment is that students with dyscalculia are the experts; only they can tell you what they know and how they think. In this chapter, we provide tools and supports to help you design, enact, and analyze a clinical interview with a student. Using a clinical interview allows you to identify areas of inaccessibility for students and understand their strengths. Both are essential to understand in order to support them in navigating through their inaccessibilities.

Building on a deep understanding of our students' strengths and difficulties, in Chapter 5, we pivot to identifying specific instructional practices that can be used to support students with dyscalculia. Because students with dyscalculia are often struggling with foundational mathematics concepts, they typically are not able to access grade-level mathematics and tend to receive at least some of their math instruction in special education. In this chapter, we explore how to productively work at the intersection of special education and mathematics education by centering special education's focus on *access* and mathematics education's focus on *sense-making*. We delineate specific access and sense-making practices that have supported students with dyscalculia to engage in grade-level mathematics even when they have not yet "mastered" the prerequisite skills.

In Chapter 6, we explore what to do when access and sense-making practices alone do not resolve the inaccessibilities experienced by students with dyscalculia. We describe tools and approaches that have been effective for addressing the specific inaccessibilities students with dyscalculia experience with quantity and notation. We describe how to work in partnership with the students to identify what approaches are effective. This returns to one central idea of this book: Students with dyscalculia are the experts. Only the students themselves know what is accessible and what tools and approaches work for them.

We conclude the book with Chapter 7, which provides a map of your exploration of dyscalculia. It highlights the big ideas in the book and offers guidance for how you can begin your own exploration into dyscalculia with your students. This exploration is a journey. We are so excited to embark on it with you, and we can promise you that it is worth it.

ACKNOWLEDGMENTS

Our author team would like to express our deepest gratitude to the students who have taught us so much about dyscalculia. This book would not be possible without you. Thank you to our editor, Debbie Hardin, whose gentle guidance helped this book take shape.

KATIE'S ACKNOWLEDGMENTS

As a dyslexic academic, I must start by thanking Clary Creagor, who taught me how to read, and Judith Farrington, who taught me how to write. I would not have this career that I love without their guidance early in my life.

Many people have meaningfully informed my understanding of disability and mathematics education. First, I would like to thank my graduate advisor, Alan Schoenfeld, for his unwavering support of me and my work. He encouraged my exploration of my own research interests and was an invaluable guide and advocate as I delved into largely uncharted waters. He allowed me to develop as a researcher and find my own identity while also providing guidance and support at every critical juncture. Second, I would like to thank Geoff Saxe, Maryl Gearhart, and Susan Schweik, who have all meaningfully shaped my understanding of mathematics education, pedagogy, and disability. You have helped me find the language and the theoretical framing to situate my exploration of dyscalculia. Third, I would like to thank those scholars whose work sits at the intersection of special education and mathematics education, including Cathery Yeh, Erica Mason, Jessica Hunt, Paulo Tan, and Rachel Lambert. I am deeply appreciative for the community and ways in which I continue to learn from each of you and your critically important work. Fourth, I am deeply grateful to all the incredible teachers who have opened their classrooms to me, in particular, Heather Christothoulou, Elizabeth Little, Amanda Stempel, and Gwen Sweeney. I have loved learning with and from you.

This book is the culmination of many years of research and writing. I am deeply appreciative of my coauthors, Amy and Dylan, who have spent countless hours diving into video data and working through the ideas that became central to this book. I will be forever grateful to Kara Jackson, Heather Cheney, Emily Shahan, Darrell Earnest, and Colleen Lewis, my writing companions, who have helped me carve out a joyful and focused writing space for these ideas to percolate and take shape. Thank you to Rachel Lambert, who thought we should write this book, and convinced us that we could.

Grant funding has made the research in this book possible. I am grateful for the funding that has supported this research over the years, including: the National Science Foundation, the National Academy of Education, The Spencer Foundation, and the University of Washington.

I could not have undertaken this journey without the support of my family. Thank you to my incredible wife, Caitlin Harren, who has always encouraged me and this work. You have always been the best partner, planner, chef, conspirator, caregiver, cheerleader, and friend. I am grateful every day for you. To Brooke and Trin, thank you for making me a parent and making us a family. In our eight years together, no one has taught me more than you. Thank you to my incredible sisters, Molly Saxelby and Colleen Lewis, who have provided just the right amount of work and fun throughout our lives.

Finally, thank you to my first teachers—my parents, Anne and Dennis Lewis, who have always supported and believed in me and this work. None of this would have been possible without you.

AMY'S ACKNOWLEDGMENTS

Thank you to my former teaching partners and colleagues for your friendship and generosity. Thinking of our time together will always make me smile.

To my current and former students, thank you for allowing me to be a part of your lives over the years. You have all taught me the importance of listening closely.

Katie and Dylan, you are both a delight to work alongside. Dylan, thank you for graciously sharing your story and allowing me to learn from you.

Katie, thank you not only for trusting me to join you on multiple writing projects, but also for being an incredible thinking partner and mentor.

Thank you to my family for their ongoing support and guidance—my wife, Lissa; my grandparents, Christine and Harley; my mother, Diane; my sister, Desirae; and my in-laws, Gail and Mark. Also, to GoGo and Lucy, for all of your much-appreciated dog cuddles.

Finally, thank you to my father, Robert, for inspiring me to pursue a career in education.

DYLAN'S ACKNOWLEDGMENTS

I'd like to acknowledge and thank the late professor George Cobb. His keen observations as my advisor led me to seek out necessary answers for myself. If it wasn't for that initial nudge, I don't know if I would've been able to keep up the momentum I needed to complete my degree.

Thank you to Katie for working with me for over a decade now. Seeing all our work culminate into this book is incredible, and I finally understand base ten and fractions because of you! Thank you to my father, who insisted I keep up with my math education no matter how hard it got. That foundation of grit and stubborn determination was the well I tapped into and continue to when things get tough. Thank you to my wonderful husband, Jeff, for listening to me talk about dyscalculia and math and writing and editing and all the things. I am excited to take everything I've learned over the years and pass it on to our two beautiful daughters and any future children we are blessed with.

PUBLISHER'S ACKNOWLEDGMENTS

Corwin gratefully acknowledges the contributions of the following reviewers:

Emily Dwivedi
Special Educator and Math Resource Teacher
Baltimore County Public Schools
Baltimore, MD

Teri Fechter
Instructional Coach, Special Education
Appleton Area School District
Little Chute, WI

James Manzano
Math Learning Specialist, Community Roots Charter School
Ridgewood, NY

Liesl McConchie
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Tammy McMorrow
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ABOUT THE AUTHORS

Katherine E. Lewis is an associate professor in the College of Education at the University of Washington. She is a researcher and educator who is fundamentally concerned with designing accessible educational contexts—particularly for students with disabilities. Her interest in dyscalculia grew out of her own experience being diagnosed with dyslexia when she was young and has informed her anti-deficit approach to understanding disability. In her research, Katie explores dyscalculia in authentic learning environments, evaluates the accessibility of instruction, and considers ways in which students may compensate. Over the past 20 years, she has worked predominantly with students with dyscalculia. In her work she attempts to understand the nature of their difficulties, honor their unique ways of thinking, and design instruction that is grounded in their access and sense-making.

Amy B. Anderson is a Seattle-based online educational therapist who specializes in supporting neurodiverse students in math and executive functioning skills through individualized one-on-one support. Before working as an educational therapist, she taught fifth-grade math and science for a decade in University Place, Washington. Amy meets her students where they are and respects them as experts on their learning. As someone with dyslexia, she understands the importance of providing a welcoming, safe, and fun environment for her students to take risks, make mistakes without judgment, set goals, and learn to self-advocate. In recent years, she has collaborated with Katie on research exploring instructional practices effective for students with dyscalculia.

Dylan M. Lynn is an educational consultant who specializes in students with dyscalculia. Her interest in dyscalculia grew out of her own experience being diagnosed with dyscalculia while she was completing her statistics degree at the University of California, Berkeley. Because the only university accommodation she was offered was a math waiver (useless for a math major), she sought out support on her own. She connected with Katie, and they embarked on a research collaboration spanning a decade. Their work aimed both to understand the inaccessibilities Dylan experienced and to document the ways in which she had compensated up to that point. Dylan graduated with a statistics degree and went on to work as a data analyst for several years before turning her focus to education. She currently works as a dyscalculia consultant and advocate with a goal to provide teachers, students, and families the resources that she never had.

STUDENTS WITH DYSCALCULIA FEATURED IN THIS BOOK

Note that all names are pseudonyms, with the exception of Dylan Lynn—an author of this book.

Destiny—A third-grade student who received daily one-on-one special education instruction in reading, writing, and mathematics. Katie worked one-on-one with Destiny on mathematics, daily, for six months during the COVID-19 school closure.

Dylan Lynn—One of the authors of this book. Dylan and Katie began working together during Dylan's senior year at UC Berkeley as she completed her statistics degree.

Emily—A student with learning disabilities and specific difficulty with math. Katie tutored Emily weekly from eighth through twelfth grade across a range of mathematical topics.

Evelyn—A third-grade student who received special education instruction in reading, writing, and mathematics. Katie observed Evelyn in her third-grade mathematics class, worked with her individually and in small groups throughout the year, and worked with her teaching team.

Lisa—A 19-year-old community college student with specific difficulties with mathematics and who had failed an arithmetic course. Katie worked with Lisa on fraction concepts during a set of 10 one-on-one fraction tutoring sessions.

Luna—A 10-year-old fourth-grade student with difficulties reading and writing numerals and solving arithmetic problems. Katie worked with Luna one time, interviewing her about her mathematical understanding over Zoom.

Melissa—A 31-year-old community college student with dyscalculia. Katie worked one-on-one with Melissa on math focused on integers and algebra. There were a total of 19 tutoring sessions spanning eight months.

Porter—A nine-year-old fourth-grade student who attended a private school specializing in serving students with language-based learning disabilities. Katie worked with Porter weekly for four months in his fourth-grade year.

Ryan—A 14-year-old eighth-grade student who attended a private school for students with language-based learning disabilities. Katie worked with him on fraction understanding during eight one-on-one sessions.

To all our students, who have been our greatest teachers.

Chapter

1

What Is Dyscalculia? Welcome to the Inquiry

Lisa, a 19-year-old college student with dyscalculia, still counted on her fingers to solve addition problems, could not add fractions, and failed a prerequisite arithmetic course at her community college. She explained: "I was never good at math. Sometimes if I have someone right there with me the entire time going through problem by problem, it can make sense. . . . But the minute I sit down to do tests, or if you ask me to do a similar, or even the same problem alone, it's like a different language. That's how I explain it to everyone. It's like I'm reading Arabic. But I don't speak Arabic." Despite her academic success in all nonmathematical domains, she continued to struggle with elementary mathematics. Her difficulties with math were localized and exceptional. She described: "It truly feels like, like a part of my brain is missing that is supposed to understand this information." Dyscalculia is a disability that results in significant difficulty processing and manipulating quantities. For the estimated 6–8% of students like Lisa who have dyscalculia, their difficulties with mathematics can limit their academic options and career prospects.

- Do you know a student like Lisa, a student who seems to understand a math concept in one moment, and not the next?
- Have you encountered a student who has profound difficulties with mathematics despite their best efforts?

You may not be familiar with the term *dyscalculia*—that's not unusual. There are several reasons why. Although dyscalculia is as common as dyslexia, there is much less research on dyscalculia, and consequently, information about dyscalculia rarely filters down to teacher training or professional development. Also, there is no test for dyscalculia. Several researchers have developed different kinds of screeners or checklists, but there is no agreed-upon assessment to determine if a student has dyscalculia. Lastly, different terminology is used across the fields of neurology, psychology, and education. In schools in the United States, "specific learning disability in mathematics" is the most common terminology used. This disability label can be given by school psychologists when students have low math scores, or when students do not respond to intervention, within a Response-to-Intervention model. Complicating matters, not all students identified with a "specific learning disability in mathematics" will have dyscalculia, and not all students with dyscalculia will be assigned this label. Although many students with dyscalculia will have low math achievement, there are a whole host of reasons why a student might struggle (e.g., insufficient instruction, lack of educational opportunity, environmental stressors, mathematics trauma, math anxiety). You may never work with a student with the designation of "dyscalculia"—but the prevalence of this disability suggests that you will have students in your class, or on your caseload, who do have dyscalculia. Because there is no diagnostic testing for dyscalculia, you may never know for sure if one or more of your students has it. The good news is that the same assessment approaches and instructional interventions that are effective for students with dyscalculia are useful for all students who are struggling.

In this chapter, we briefly review the complexities of the unclear diagnostic criteria for dyscalculia and discuss the ways in which this is further complicated by dyscalculia's co-occurrence with other disabilities. We describe the characteristics of dyscalculia, provide a brief overview of some possible underlying neurological causes identified in research, and explore how these cognitive differences can lead to unique patterns of errors. Unlike in most research, we take an anti-deficit approach to understanding this disability. We illustrate how careful listening can provide far greater insight into understanding both the difficulties students experience, as well as how best to support them.

HOW DO YOU TELL IF A STUDENT HAS DYSCALCULIA?

Perhaps you have been told by a pediatric neurologist or a school psychologist that a student in your class has dyscalculia. You may have a student with a math goal on their IEP and wonder if they might have dyscalculia. You could also have a student who has no identified disability but is struggling with math so profoundly, you wonder about dyscalculia. Unfortunately, defining exactly what dyscalculia is and who has it is not a straightforward process. This is a problem in both research and practice.

The Diagnostic and Statistical Manual of Mental Disorders (5th ed.; *DSM–5*; American Psychiatric Association, 2013) defines dyscalculia as a specific learning disorder in mathematics (see Figure 1.1).

Figure 1.1 • *Definition of dyscalculia from the* DSM–5

Definition of Specific Learning Disorder in Mathematics (Dyscalculia)

Student must:

- Demonstrate a history of difficulties in mathematics (at least 6 months)
- Have low achievement in mathematics compared to peers
- Have sufficient educational opportunity

Student may:

- Have difficulty understanding numbers, magnitude, and relationships
- Count on fingers rather than retrieving math facts from memory
- Get confused when solving problems and switch procedures
- Have difficulty applying math concepts, facts, or procedures to solve problems

Student must not:

- Have another primary disability
- Experience adverse environmental conditions like:
  - Economic disadvantage
  - Insufficient instruction
  - Difficulties with language fluency

SOURCE: *Diagnostic and Statistical Manual,* 5th ed.

Students with dyscalculia have difficulty with number sense, memorization of math facts, accurate and fluent calculation, and math reasoning. The *DSM–5* criteria require that students demonstrate a history of difficulty despite sufficient educational opportunity. The student must have low math achievement compared to their peers, and these difficulties cannot be attributed to another disability or adverse environmental conditions like economic disadvantage or language fluency. In educational research, there are a plethora of terms used to refer to dyscalculia: mathematics learning disability, arithmetic disability, math disability, mathematics difficulty, or learning disability in mathematics. Researchers still disagree about exactly what dyscalculia is and the best way to identify students with it (Lewis & Fisher, 2016). Because there is no test, researchers often use low math achievement test scores to identify students with dyscalculia (e.g., students scoring below the 10th or 25th percentile). Unfortunately, the use of different tests and different criteria to determine which students have dyscalculia makes it very difficult to compare findings across studies (Lewis & Fisher, 2016). In addition, researchers are rarely able to determine if the student's low achievement test score is due to dyscalculia or to environmental factors (e.g., insufficient educational opportunity). It is not surprising that many studies report somewhat different findings and that often the findings mirror patterns of difficulties similar to the achievement measures used to identify students in the first place.

CO-OCCURRENCE WITH OTHER DISABILITIES

Complicating the issues of dyscalculia identification, dyscalculia can, and often does, occur with other disabilities. For example, researchers estimate that between 30–70% of students with dyscalculia will also have dyslexia (Wilcutt et al., 2013). Similarly, researchers have found that 36% of students with dyscalculia also have dysgraphia (disability in writing; Ashraf & Najam, 2020). It is unclear if this co-occurrence of disability is because there are similar underlying cognitive mechanisms for the disabilities, or because reading and writing difficulties may lead to difficulties accessing and learning mathematics (Swanson et al., 2009). Still, other studies have identified higher rates of Attention Deficit Hyperactive Disorder (ADHD), autism, and social-emotional and behavior disorders in students with dyscalculia (Bergen et al., 2023; Desoete, 2008; Gross-Tsur et al., 1996; Morsanyi et al., 2018). Unfortunately, assessments may fail to be sensitive enough to differentiate dyscalculia from other disabilities, because dyslexia, dysgraphia, autism, ADHD, or even math anxiety can cause students to score low on a mathematics achievement test (Krinzinger, 2019). Indeed, one of the authors, Dylan Lynn, was misdiagnosed with ADHD because of her performance on number-related working memory tasks. Although she was prescribed and started taking Adderall, it did nothing to improve her ability to process numbers, indicating that ADHD was not the cause of her difficulties.

It may be overwhelming to think of disentangling the impact of each of these disabilities. However, it is not necessary—or even useful—to definitively determine the "right" disability diagnosis. Students in your class may have a variety of different kinds of disabilities that all may be impacting their math learning and performance. Fortunately, the instructional strategies and approaches in this book are student-centered and responsive to the individual student. These approaches will be effective not only for students with dyscalculia, but also for students with a range of disabilities who struggle with math. In the next section, we focus specifically on the characteristics identified in students with dyscalculia and how research has focused on understanding the underlying causes.

RESEARCH ON DYSCALCULIA

Despite the overwhelming complexities with diagnostic criteria, we have some of the best insight into dyscalculia thanks to studies of extreme cases (adults with significant, persistent, and localized difficulties, like Lisa's) or longitudinal studies, which follow students over many years (Mazzocco & Räsänen, 2013; Price & Ansari, 2013). Both of these methodological approaches enable researchers to disentangle dyscalculia from other potential disabilities and environmental impacts. These kinds of studies have provided greater insight into identifying the characteristics that are unique to dyscalculia and the possible underlying causes. Beyond the broad characteristics identified in the *DSM–5*, researchers have identified specific areas of difficulty for students with dyscalculia.

Students with dyscalculia may have difficulty doing the following:

- **Counting**—particularly from a number other than 1, or backward
- **Ordering and comparing numbers**—quickly ordering numbers under 10, and using place value to order whole numbers, decimals, and fractions
- **Remembering and quickly recalling math facts**—quickly and fluently retrieving answers to problems like $4 + 5 =$ or $3 \times 7 =$
- **Reading a clock**—reading the hands of a clock, or making judgments about durations

These difficulties are well-documented in students with dyscalculia, and many of these issues continue into adulthood. It may be that you know one or more students who have significant difficulties with the mathematical tasks listed above. Although all students may struggle with these topics when first learning them, the persistence of these difficulties seems to differentiate students with dyscalculia from students who struggle with math but do not have dyscalculia. Students with dyscalculia experience persistent difficulties with mathematics, and often make errors that are unlike those made by their peers (Lin et al., 2025).

Diving Into the Research

Persistent Difficulties on the "Easiest" Problems

Because accurate identification of dyscalculia is such a critical issue in the field, Michèle Mazzocco and colleagues embarked on longitudinal research. Instead of relying upon one math achievement score to classify students, they collected data with students from kindergarten through eighth grade—assessing them every year (Mazzocco & Myers, 2003). They recruited 249 kindergarten students from 23 classes and specifically selected schools to minimize the likelihood of environmental factors impacting students' math performance. The schools they recruited from had low mobility and low rates of free and reduced lunch participation. Students met the criteria for dyscalculia only if they scored below the 10th percentile on the math subtests of the Woodcock-Johnson Test of Achievement (Schrank et al., 2014) over several years (third–sixth grade). Because of the wealth of data they collected over nine years, they were able to differentiate students with dyscalculia from students with low mathematics achievement, who did not have dyscalculia. This enabled them to compare students with dyscalculia to students who were either typically achieving or low achieving. What they found was critically important. Students with dyscalculia do not just lag behind their peers; they make fundamentally different kinds of errors. For example, they found that eighth-grade students with dyscalculia often made errors on "easy" fact problems (e.g., for numbers less than 5). They also made more **operand** errors (e.g., $3 \times 4 = 15$), and their errors were more often decade-inconsistent (e.g., $4 \times 6 = 32$)—meaning the tens digit did not match that of the correct answer (Mazzocco et al., 2008). This study suggests that students with dyscalculia may make *qualitatively different kinds of errors* than their low-achieving and typically achieving peers. Indeed, other studies have identified qualitative differences in the errors students with dyscalculia make across topic areas including: arithmetic (Mazzocco et al., 2008), fractions (Lewis, 2014, 2017; Lewis & Lynn, 2018; Mazzocco & Devlin, 2008; Mazzocco et al., 2013), integers (Lewis et al., 2020), and algebra (Lewis et al., 2022).

Operand is a number or quantity that is used in a mathematical operation problem.

Proposed Causes of Dyscalculia

Most research on dyscalculia has been focused on trying to identify the cognitive causes of dyscalculia. The assumption is that if the causes are definitively identified, then practitioners can better know how to intervene. There are two dominant perspectives about the origin of dyscalculia. Some researchers believe that dyscalculia is due to differences in how students process numbers. They argue that dyscalculia results from *domain-specific* differences in the brain systems that are responsible for perceiving and manipulating numbers (Butterworth et al., 2011). Because students with dyscalculia process numbers differently, they will have difficulty with tasks involving numbers or quantities, and it will impact their math learning. Other researchers believe that dyscalculia is due to *domain-general* cognitive skills that are not number specific, like memory, executive function, or attention (Geary, 2004; Szucs et al., 2013). These are referred to as domain-general explanations—because memory and attention would impact the student across a range of topics—perhaps partially explaining the co-occurrence of dyscalculia with other disabilities.

Both domain-specific and domain-general hypotheses offer plausible explanations for the difficulties experienced by students with dyscalculia. Research supporting each perspective has used measures of speed, accuracy, and capacity to document that students with dyscalculia have "deficits" (see Table 1.1). These two perspectives focus on contrasting areas of cognitive processing (i.e., number processing vs. executive function). Although these hypotheses provide conflicting explanations for dyscalculia, more recently researchers have speculated that dyscalculia may be due to some combination of both domain-specific and domain-general causes (Kaufmann et al., 2013; Mazzocco & Räsänen, 2013).

Table 1.1 • The difference between domain-specific and domain-general explanations for dyscalculia

Features	Domain-Specific	Domain-General
Definition	Differences in brain systems that are specifically responsible for processing quantitative information.	Differences in broad cognitive skills, which impacts performance in multiple areas, including math.
Key areas affected	Number processing, number sense, quantitative reasoning.	Attention, working memory, and executive function.
Measures	Speed and accuracy on quantity comparison or numerical judgment tasks (e.g., dot comparison).	Accuracy and capacity for attention, working memory, and executive function.

(Continued)

(Continued)

Features	Domain-Specific	Domain-General
Impacts on math	Associated difficulties • reading/writing numbers • comparing, ordering, and manipulating numbers • remembering numerical information (e.g., math facts)	Associated difficulties • manipulating and holding information in working memory • storing information in long-term memory • organizing and attending to information
Impacts beyond math	Difficulties expected for other topics that require numerical manipulation (e.g., science).	Difficulties expected across all academic subjects.

Limitations of Dyscalculia Research

Both domain-specific and domain-general cognitive differences may contribute to the difficulties that students with dyscalculia experience. However, we find this research to be of limited use for teachers. One issue with both domain-specific and domain-general research is that it tends to focus exclusively on measures of speed and accuracy. If the only issues facing students with dyscalculia were about speed and accuracy, these difficulties would be solved simply by giving them a calculator. Unsurprisingly, this is not the solution. Students with dyscalculia continue to struggle with mathematics, even when using a calculator.

Dyscalculia Myth Debunked

Myth: We can understand dyscalculia by measuring students' speed and accuracy, because those skills are essential for mathematics competency.

Debunked: Mathematics competency is about far more than speed and accuracy. Although much of the research on dyscalculia exclusively focuses on these kinds of quantitative measures, they tell us very little about what the student can do and why the student is struggling. Students with dyscalculia continue to struggle, even when they have access to a calculator, a tool that can quickly and accurately compute. This suggests that to understand dyscalculia, we may need to understand more about what and how the students are reasoning, rather than simply documenting their speed and accuracy performance.

Disrupting This Myth: Students often believe that whoever is the fastest at math is the best at math. To help disrupt this in your classroom, you can change how you choose and launch tasks to help students focus on deep engagement with the content rather than quick answers. You might ask students to come up with multiple ways of solving one problem, or praise students for their perseverance and focus on one problem rather than completion of assignments.

We argue that what is often being measured and analyzed in most research on dyscalculia (e.g., students' speed and accuracy) tells us very little about how to support students. Even error analysis, which provides some interesting insight into the unique patterns of errors made by students with dyscalculia, cannot explain why those patterns emerge or provide guidance about what to do when students make those errors. Most researchers are not studying the learning process directly; therefore, they do not capture learning difficulties as they emerge. Instead, they infer learning difficulties based on students' performance on tests. A low score on a test is understood to reflect their prior learning (or lack thereof). They do not explore why students may have these difficulties, or how these difficulties emerge and may impact subsequent learning. Therefore, it remains unclear why students with dyscalculia have persistent difficulties in almost all domains of mathematics and how dyscalculia will impact their ability to learn and do mathematics. Teachers are left with unclear guidance and scant recommendations for how to identify and support their students with dyscalculia.

Challenges for Teachers

- No test for dyscalculia
- Unclear diagnostic criteria
- Co-occurrence with other disabilities
- Lack of instructional recommendations for both domain-general and domain-specific explanations
- Lack of explanation and instructional recommendations for unique error patterns

BEYOND CAUSES: LEARNING FROM STUDENTS

In this book, we take a different approach. Rather than focusing on errors students make on tests, we draw upon research that explores the experiences of students with dyscalculia during their attempts to learn and do mathematics. Throughout the book, we will rely heavily upon examples that were taken from video recordings of students in the process of learning and problem-solving. We are interested not only in the student's answer, but also in how and why they arrived at that answer. We believe that these records of practice provide essential insights into the nature of the students' difficulties and the complexities that emerge when attempting to meaningfully support students. Through working closely with dyscalculic students, we can learn more directly about the difficulties they experience and what underlies those difficulties. To illustrate the power of this kind of in-depth view of student learning, we provide several examples taken from one-on-one tutoring sessions with a student with dyscalculia, Emily (Lewis, 2009b). We focus on her understanding of multiplication facts because prior research has suggested that this is a hallmark of dyscalculia—and longitudinal research has documented the unique errors made by students with dyscalculia.

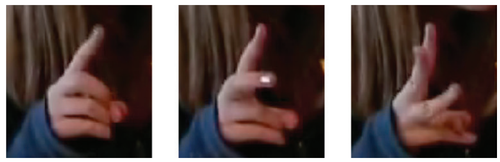
Learning From Emily

I (Katie) tutored Emily weekly in mathematics from eighth through twelfth grade. Emily's eighth-grade teacher identified her as a student who had been labeled as having learning disabilities within the school system and had persistent difficulties with mathematics. Emily met dyscalculia criteria and formal learning disabilities testing placed her below the 10th percentile on several measures of mathematics achievement. Over the course of the five years I worked with Emily, I saw that she experienced many of the difficulties found in students with dyscalculia, including the characteristic difficulty with math facts. She still relied on finger counting for adding, and no matter how many flashcards or math programs she practiced, she could not memorize discrete multiplication facts. She was able to use finger counting and other strategies when solving simple operations, given sufficient time. However, her difficulty with fluent retrieval of math facts was evident when she started working with more complex math problems (Lewis, 2009b).

At times, Emily would practice math facts using a computer program. Analysis of Emily's multiplication math fact answers found a pattern of errors similar to those documented in students with dyscalculia (Mazzocco et al., 2007). Most of her errors (75%) were operand errors (solving the problem with the wrong operand) and 79% of her errors were decade-inconsistent (answers in which the digit in the tens place was wrong, and consequently the answer was farther away from the correct answer). Because I was interested in exploring why this pattern of errors emerged, I inquired into Emily's thinking about multiplication directly.

During one session, I asked Emily to explain how she solved the problem “7 × 3 = .” Emily replied:

“You like count up 7 14 21”



Emily was not retrieving the answer of 21 directly from long-term memory; instead, she was iterating through a list of multiples of 7, while keeping track of her position in the list on her fingers. Emily memorized all her multiplication facts as lists of multiples that she would recite. For example, she referred to her “nines” as: “9, 18, 27, 36, 45, 54, 63, 72, 81.” When using a particular list, the recitation was rapid and generally completed in less than five seconds. Emily did not, or was not able to, memorize her multiplication facts as discrete elements. We understand this atypical way of memorizing facts to reflect an underlying cognitive difference. It was less important to uncover whether Emily’s cognitive difference was because of a domain-specific number processing issue or a domain-general memory issue. What was more important to understand was that she had created an unconventional way of memorizing math facts.

We understand this kind of student-invented alternative as a compensation. Compensation is a big theme throughout this book. We understand compensation to be an individual’s deliberate action when they encounter an inaccessible task or expectation. For example, teachers expect that eighth-graders have fluency with their math facts, and that they can use these math facts when solving more complex problems. Emily encountered inaccessibility because she was asked to solve problems that required this fluency. She compensated for this inaccessible task demand by developing an alternative approach for herself. Emily memorized a list of numbers in the same way that one might memorize the letters of the alphabet or other ordered lists (e.g., planet names, presidents, etc.). In general, counting up multiples was an effective strategy for Emily in the context of solving simple multiplication problems. Emily was efficient in using this strategy, in that she often chose to access the multiples list for the larger operand regardless of the order of the numbers in the problem (e.g., accessing her sevens rather than threes to solve 3 × 7 =). This preference for using the larger operand was so strong that she even used it in contexts where it required more work. For example, when she was attempting to solve the problem "What times 2 is 16?" she did not access her twos list. She initially thought there was no answer. She explained, "You can't do it . . . because 2 doesn't go into 16 evenly." When I asked if she was sure, she answered, "Yeah, 5, 10. 6, 12. 7, 14. Oh! 8, duh!" To solve this problem, she used a guess-and-check approach with several larger operands, accessing the second element of her fives, sixes, sevens, and eights, rather than iterating through her twos. Emily's preference for accessing the multiples list for the larger operand was so strong that she did this even when it required more effort.

Emily's unconventional way of remembering and accessing her multiplication facts helps explain the unique errors documented in other students with dyscalculia. As with other students with dyscalculia, Emily's errors were most often operand errors. An operand error (solving a problem with the wrong operand) could occur any time Emily miscounted in her recitation of her multiples list, or any time she omitted a number from that list. Indeed, 60% of her operand errors occurred when the operand was one more or less than the correct operand (e.g., for $7 \times 3 =$ answers of 14 or 28). Second, Emily's tendency to use the multiples list of the larger operand would lead to a higher probability that miscounting would result in a decade-inconsistent error. Both of these errors—operand errors and decade-inconsistent errors—have been documented in students with dyscalculia, but not understood (Mazzocco et al., 2008). Careful listening and observation of Emily's process provide an explanation for these patterns of errors in her case.

This detailed exploration also identified several *other* patterns of errors that were explained by Emily's reliance upon her multiples list. Sixty-seven percent of errors Emily made on math fact problems involved problems where one of the operands was either a 6 or a 9. Because Emily often used her multiples lists by reciting them under her breath, the reasoning for this pattern was not clear until one day when Emily was attempting to list multiples of 6 to find a common denominator for a fraction problem. In this excerpt, she switched between her sixes multiples list and her nines multiples list four separate times (for ease of reading, we have indicated this switch with red text and italics).

> **Katie:** Let's start listing out some multiples of 6.
>
> **Emily:** 6, 12, 18, 24, *36, 45, 54, 63, 72, 81*.
>
> **Katie:** (*writing* "6, 12, 18, 24, 36")
>
> . . .
>
> **Katie:** Do you do it in your head 6 times 1, 6 times 2, 6 times 3?
>
> **Emily:** (*shakes head no*) I just memorize them, and I don't even know if it's right. 6, 12, 18, 24, *36, 45, 54, 63, 72, 81*. Is that right or wrong?

> **Katie:** I don't know—
>
> **Emily:** 6, 12, 18, 24, *36, 45, 54, 63, 72, 81*? Or are those nines?
>
> **Katie:** No those are sixes (*pointing to the page*).
>
> **Emily:** 9, 18, 27, 36, 45, 54, 63, 72, 81. No, they are nines.
>
> **Katie:** Uh huh.
>
> **Emily:** 6, 12, 18, 24, *36, 45, 54, 63, 72, 81*. I'm confused. They are both nines.

In this instance, Emily recited her sixes on four separate occasions, but in each instance, she shifted from her sixes to her nines. In an attempt to correct herself, she then listed out her nines and was further confused, not knowing if she was listing sixes or nines or if both strings of numbers were multiples of nine. The linguistic similarity between the sixes and nines multiples list (see Figure 1.2) provides further insight into this additional unique error pattern that emerged in Emily's math fact data.

Figure 1.2 • *The linguistic similarities between parts of the sixes and nines multiple list*

Sixes:	**eighteen**	**twenty-**four	**thirty**
Nines:	**eighteen**	**twenty-**seven	**thirty-**six

This linguistic similarity can explain why Emily may have accidentally shifted from one list to the other (see Figure 1.3), and helps explain why 67% of the errors she made in the math facts program involved a problem with a 6 or 9 operand.

Figure 1.3 • *How Emily switched from her sixes to her nines multiples list*

Emily: "6, 12, 18, 24, 36, 45, 54, 63, 72, 81"

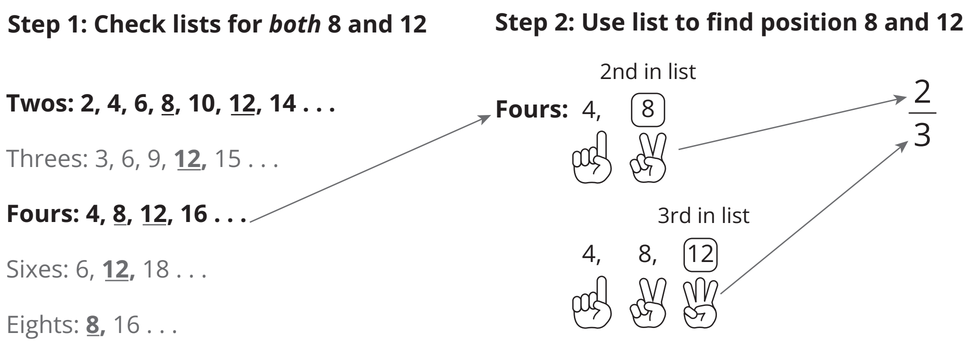
Sixes: 6 12 18 24 30 36 42 48 54

Nines: 9 18 27 36 45 54 63 72 81

Not only is Emily's strategy subject to unique vulnerabilities of linguistic similarity, but her multiples list strategy created additional cognitive demands, particularly when working to solve more complex math problems.

This additional cognitive demand of Emily's multiples list was evident when working with fractions. To illustrate this, consider how she would simplify a fraction like $\frac{8}{12}$. First, she would need to determine what multiples list had both 8 and 12 on it. She might access several different multiples lists with either 8 or 12 to check for the presence of both (see Figure 1.4). Next, if she decided that the fours list has both 8 and 12, she then would need to determine the ordinal position of 8 and 12 in that list. She would count up by fours ("4, 8"), while keeping track of her position on the list with her fingers. Because 8 was the second number in her list, the simplified fraction would have a numerator of 2. She then could use the list again to determine the ordinal position of 12: "4, 8, 12" (see Figure 1.4). Because 12 was the third number, the denominator of the simplified fraction would be 3. Using her multiples list was cognitively demanding because she needed to access several multiples lists, iterate through the list at least twice, and identify the ordinal position of the original numerator and denominator within the list.

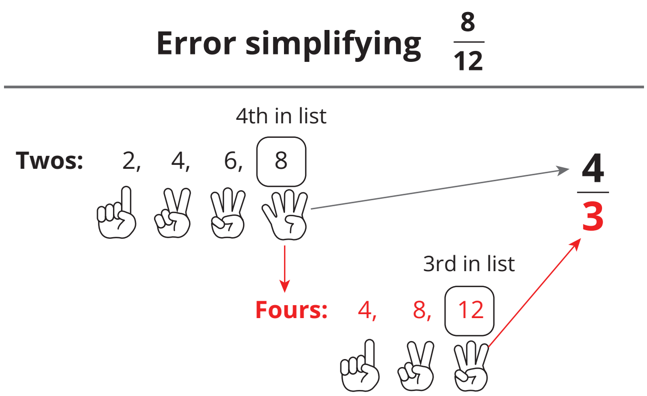
Figure 1.4 • *Emily's approach to simplifying fractions with her multiples lists*



ICON SOURCE: istock.com/nanamenico

Understanding the different ways Emily stored and accessed multiplication facts also enabled us to understand her unconventional errors simplifying fractions (Lewis, 2007). For example, Emily often incorrectly simplified $\frac{8}{12}$ to be $\frac{4}{3}$ or $\frac{2}{6}$. These reoccurring and unconventional errors can be understood when we consider her process. Imagine that Emily correctly determined that her twos list contained both 8 and 12. She would simplify the numerator by iterating through her twos (2, 4, 6, 8) until she gets to 8, the 4th number (see Figure 1.5). At this point in time, she would be thinking of the number 4 (and perhaps writing down "4"), and instead of accessing her twos list again to determine how 12 should be simplified, she might access her fours (4, 8, 12) to determine that 12 is the 3rd number on that multiples list, resulting in the error of $\frac{4}{3}$ (see Figure 1.5).

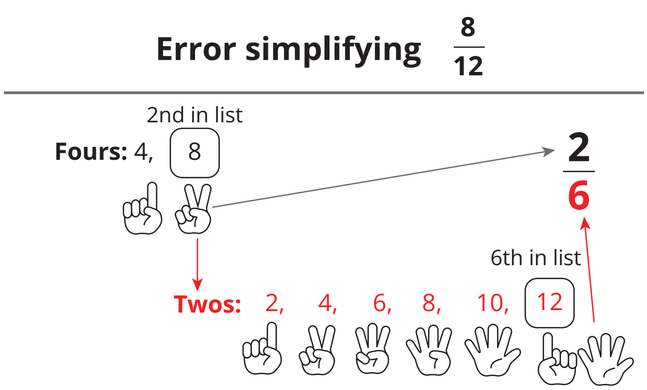
Figure 1.5 • *How Emily's use of her multiples lists may have led to her common error of simplifying $\frac{8}{12}$ to $\frac{4}{3}$*



ICON SOURCE: istock.com/nanamenico

This same logic holds for Emily's second unconventional answer of $\frac{2}{6}$. If she accessed her fours list to determine that 8 was the second item in the list, her twos list may have been inadvertently cued. If she then accidentally accessed her twos list, she would end up with the fraction $\frac{2}{6}$ (see Figure 1.6).

Figure 1.6 • *How Emily's use of her multiples lists may have led to her common error of simplifying $\frac{8}{12}$ to $\frac{2}{6}$*



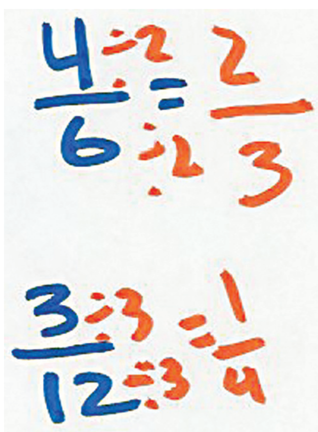
ICON SOURCE: istock.com/nanamenico

By carefully watching and listening to the student engage in mathematical problem-solving and learning, we can gain clearer insights into why students may make particular kinds of unusual errors. When we investigate how the student is solving and making sense of the problems, we can understand both why these errors might emerge and the additional cognitive strain some of their strategies place on them.

Supporting Students

This detailed exploration of Emily's strategies and their challenges enabled me to consider more appropriate ways to support Emily. She would not benefit from extra time spent drilling her on math facts; this approach had been failing since elementary school. Instead, I noted that the kinds of errors Emily made had to do with slipping from one multiples list to another. To address this difficulty when simplifying fractions, I provided supports to help prevent her from switching lists. The instructional approach simply involved asking Emily to write down the factor by which she was dividing both the numerator and denominator (see Figure 1.7). Analysis showed that this approach increased her accuracy on these problems (Lewis, 2007). Before this approach, she had 70% accuracy on fraction simplification problems; after its introduction, she had over 93% accuracy when she wrote down the factor. This approach also increased the number of problems she was able to solve in a session.

Figure 1.7 • *The instructional access approach that involved Emily writing down the factor by which she was simplifying the fraction*



EXPLORING DYSCALCULIA THROUGH THE DETAILS

There are several important things to note in these examples. First, we acknowledge that there are some things that Emily's brain was not set up to do—memorizing math facts was one of them. She has a cognitive difference. We don't believe there is much use in belaboring what underlying neural pathways contribute to this difficulty. The results are the same: Emily experienced persistent and ongoing difficulty with her math fact fluency. Second, we acknowledge that math instruction often presumes that students have memorized addition and multiplication facts. For example, when students are asked to simplify fractions, it is assumed that they have fluency with their basic facts. Instructional approaches in schools often take this kind of prerequisite knowledge as a given. This means that much of the mathematics Emily encountered during her middle and high school years was at least partially **inaccessible**. Inaccessibility results when the student's natural cognitive processing does not align with the expectations and task demands of formal school mathematics. Third, Emily created an alternative approach, a **compensation**. Emily's multiples list compensation circumvented the kinds of memorization that were not possible for her, but were expected of her by task demands and teachers. This alternative approach was effective in many contexts and enabled her to work through problems she would not otherwise have been able to solve. However, this compensation was also cognitively more demanding, and it led to atypical errors as Emily attempted to solve more complex problems, such as fraction simplification. By carefully watching and listening to how Emily understood and solved multiplication problems, we were able to understand Emily's unique patterns of errors and design an alternative approach (e.g., writing down the factor of simplification) to provide her with increased access.

Inaccessible means that something is not accessible because of barriers resulting from designs or spaces that do not allow for the full range of human sensory, cognitive, and physical variation.

Compensation is an individual's deliberate action to gain access to otherwise inaccessible spaces, contexts, or content.

Dyscalculia Myth Debunked

Myth: Students with dyscalculia will never be successful in high school, college, and STEM careers.

(Continued)

(Continued)

Debunked: Although students with dyscalculia encounter a multitude of inaccessibilities and barriers, many nevertheless persevere and are successful in college and beyond. Despite having dyscalculia, Dylan Lynn majored in statistics and went on to work as a data analyst. She needed to learn how to navigate the inaccessibilities of the dense statistical notation and the arbitrary barriers like timed exams. Like Dylan and many other adults with dyscalculia, students with dyscalculia can succeed in school and STEM careers. We need to ensure that students with dyscalculia do not internalize deficit-based notions about their capabilities or future. As Dylan explained, "They don't say kids with dyslexia won't learn to read . . . they shouldn't say kids with dyscalculia won't learn math."

Disrupting this myth: One approach to helping students with dyscalculia realize their own capabilities is by removing arbitrary constraints and expectations. For example, avoiding timed tests and removing requirements for rote memorization can provide opportunities for students with dyscalculia to demonstrate what they are able to do in mathematics. By explicitly valuing the neurodiversity of all human beings, we can acknowledge that not everyone will do math in exactly the same way. There is incredible power when students learn what works for them.

Students with dyscalculia:

- May not be able to process or remember numerical information in the same way as their peers
- May invent creative and ingenious work-arounds for the difficulties they encounter
- May use strategies that require more cognitive load
- Often are working harder than their peers
- May make unexpected and unique errors because they are solving problems differently
- Are most successful when instructional approaches honor and work with their strategies while addressing the inaccessibility

The ways we traditionally support and expect students to do math may not work for students with dyscalculia. The barriers that students encounter can best be understood in terms of inaccessibilities. For example, Emily ran into an inaccessibility because she was not able to memorize her math facts discretely and could not quickly and reliably retrieve the answers. This inaccessibility impacted her ability to factor, simplify fractions, and divide. In this book, we explore how we can learn about these kinds of inaccessibilities by carefully watching how students solve problems and listening to their reasoning. Observing Emily's process and explanations revealed she understood multiplication differently. She created an approach, or a compensation, that worked for how she processed numbers—her multiples lists. Students with dyscalculia will often come up with ingenious work-arounds when they encounter inaccessibilities. In this book, we explore dyscalculia by diving into the details and learning from the experiences and brilliance of students with dyscalculia. These insights enable us to understand the unique difficulties students with dyscalculia experience and consider meaningful ways to support them.

This exploration into the mathematical world of our students is an active and ongoing process. In this book, we will not be able to document every difficulty a student with dyscalculia could experience or all the ways in which they compensate. Instead, we hope that by seeing examples, you can begin to hone your own observation skills. Students with dyscalculia may have unique kinds of difficulties and ways of thinking that do not align with what you know or expect of students. Our job as educators is to get curious and explore our students' mathematical worlds. We need to discover what is inaccessible, honor what they are doing, and collaborate with them to help them navigate those inaccessibilities.

EMBARKING ON YOUR OWN EXPLORATION

If you want to play around with these ideas in your own class tomorrow, here are some questions to help you dive into your own exploration:

- Does your student struggle to solve arithmetic "fact" problems?
- How are they solving these arithmetic problems?
- Are they counting all when they are calculating small sums?
- Do they confuse operator signs?
- Do they use their fingers when solving problems?
- What kinds of errors do they make?
- Are any of these errors unexpected or unusual?
- How might their difficulties impact their access to grade-level content?
- What supports might you try to help them address these barriers?

Summary

This chapter has been an invitation into the exploration of dyscalculia. Because of the ambiguity of dyscalculia criteria and identification, we can learn more by diving directly into our students' mathematical worlds. Careful listening and inquiry can reveal the inaccessibilities the student experiences, how they may be compensating, and why they may have persistent difficulties and make unusual errors. In this book, we will learn about dyscalculia by learning from the experts, the students themselves.

Reflection Questions:

- What kinds of unusual errors have you seen students make in mathematics?
- What types of questions might you ask them to understand where those errors might have come from?
- What sorts of alternative strategies have you seen your students use to solve mathematics problems?
- What might the limitations of these strategies be?
- What kinds of instructional modifications have been helpful in the past?