MATHEMATICS EXPLAINED FOR PRIMARY TEACHERS

7TH EDITION

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DEREK HAYLOCK

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Marketing manager: Lorna Patkai
Cover design: Wendy Scott
Typeset by: C&M Digitals (P) Ltd, Chennai, India
Printed in the UK

Library of Congress Control Number: 2023939382

British Library Cataloguing in Publication data

A catalogue record for this book is available from
the British Library

ISBN 978-1-5296-2630-8
ISBN 978-1-5296-2629-2 (pbk)

TABLE OF CONTENTS

ABOUT THE AUTHOR

Derek Haylock worked for over 30 years in teacher education, both initial and in-service. He was Co-Director of Primary Initial Teacher Training and responsible for the mathematics components of the primary programmes at the University of East Anglia (UEA), Norwich. He has taught mathematics at every level and, in particular, has considerable practical experience of teaching and researching in primary classrooms. His commitment to mathematics being learnt with understanding, relevance and enjoyment has underpinned his work as a writer, consultant and professional speaker. As well as his extensive publications in the field of education, he has written seven books of Christian drama for young people and written and composed a Christmas musical (all published by Church House/National Society).

OTHER BOOKS IN EDUCATION BY DEREK HAYLOCK

Browne, A. and Haylock, D. (eds) (2004) *Professional Issues for Primary Teachers*. London: Sage Publications.

Haylock, D. (1991) *Teaching Mathematics to Low Attainers 8–12*. London: Sage Publications.

Haylock, D. (2001) *Numeracy for Teaching*. London: Sage Publications.

Haylock, D. (2024) *Student Workbook for Mathematics Explained for Primary Teachers*, 7th edn. London: Sage Publications.

Haylock, D. and Cockburn, A. (2017) *Understanding Mathematics for Young Children*, 5th edn. London: Sage Publications.

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Haylock, D. and McDougall, D. (1999) *Mathematics Every Elementary Teacher Should Know*. Toronto: Trifolium Books.

Haylock, D. with Thangata, F. (2007) *Key Concepts in Teaching Primary Mathematics*. London: Sage Publications.

Haylock, D. and Warburton, P. (2013) *Mathematics Explained for Healthcare Practitioners*. London: Sage Publications.

ABOUT THE CONTRIBUTOR

Martin Smith has worked at the University of East Anglia since January 2014 and is currently Course Co-Director on the Primary PGCE course with a shared responsibility for the mathematics and physical education components. Prior to this role he taught across Key Stage 2 in and around Norwich, finding ways to break down barriers to the enjoyment of maths for both children and their adults. Martin has long since enjoyed board games, puzzles and problems and if there is a mathematical element to these, all the better. Outside of education, Martin has recently rediscovered his love of dinghy sailing and manages to combine this with watching and playing football, and spending time with his family.

ACKNOWLEDGEMENTS

My thanks and genuine appreciation are due to the many trainee teachers and primary school teachers with whom I have been privileged to work on initial training and in-service courses in teaching mathematics: for their willingness to get to grips with understanding mathematics, for their patience with me as I have tried to find the best ways of explaining mathematical ideas to them, for their honesty in sharing their own insecurities and uncertainties about the subject – and for thereby providing me with the material on which this book is based. I also acknowledge my indebtedness to James Clark and the team at Sage Publications for their unflagging encouragement and professionalism, and to Ralph Manning who has contributed the website activities for inclusion in lesson plans that are referenced in each of Chapters 6 to 28.

I acknowledge especially the significant contribution to this seventh edition of Martin Smith, Lecturer in Mathematics Education and Co-Director of the Primary PGCE in the School of Education and Lifelong Learning at the University of East Anglia, who has provided advice on recent changes in emphasis in classroom practice and research in primary mathematics. His insights and suggestions have been an invaluable contribution to the seventh edition of this book.

I dedicate this new edition to the memory of my dear wife, Christina, who died in 2022. Without her constant support and encouragement over so many years I could never have achieved what I have as an author in education.

GUIDED TOUR

This book contains a range of features and additional resources to support your learning and understanding. Here is an overview of what you will find inside the book and as part of the online resources https://study.sagepub.com/haylock7e.

FEATURES IN THE BOOK

- **Learning and teaching points** highlight specific points from each chapter for use in teaching.
- **Research focus** sections explore contemporary and seminal academic research on relevant chapter topics.
- **Self-assessment questions** at the end of each chapter give you a chance to test your learning immediately. Check your work against the answers in the back of the book.
- **Links to the Student Workbook** direct you to further self-test maths questions in the Student Workbook (available separately).
- **Glossary of key terms** in each chapter explains important mathematical vocabulary in straightforward language.
- **Suggestions for further reading** highlights more specialist reading material related to topics in each section of the book.

ONLINE RESOURCES

The following online resources (available at: https://study.sagepub.com/haylock7e) are all sign-posted at relevant points in chapters so you can test your own mathematical knowledge as you work through the book.

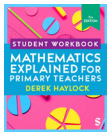
- **Lesson plans and worksheets** (created by Ralph Manning) give you lesson ideas and ways of approaching concepts for your classroom teaching.
- **'Problem solved!' animated videos** show you how to work through different maths methods to close gaps in your understanding.

- **Curriculum links** highlight where chapter content meets national curriculum requirements in England, Scotland and Wales.
- **Interactive self-assessment questions** offer a further range of multiple-choice testing with feedback on your answers.
- **Knowledge checks** offer a deeper dive on specific mathematical topics and tasks.
- **Section introduction videos** give an overview of each section of the book.

Access the online resources here: https://study.sagepub.com/haylock7e.

NEW TO THIS SEVENTH EDITION

- The Research Focus features and other references have been updated where appropriate to discuss a greater amount of contemporary research.
- Further Reading suggestions at the end of sections have been updated to include more recent relevant literature.
- Expanded content on the development of counting among young children has been brought into Chapter 6.
- Further areas of additional content across the book include: deeper discussion of arrays and technical terms now in greater use, such as subtrahend and minuend in reference to addition and subtraction.
- A new section on the external angles of triangles.
- Increased references to classroom resources and pedagogical tools that support mastery teaching in mathematics, including tens-frames, Dienes blocks and bar-modelling.
- Additional and updated references to curriculum differences across the UK, with reference to curricula documentation from England, Scotland, Wales and Northern Ireland.
- The use of 1p, 10p and £1 coins to demonstrate place value has generally been replaced by place-value counters, given the decreasing use of 1p, 2p, 5p coins and coins in general, in daily life.
- A number of the Learning and Teaching Points have been updated and a number of new ones added.
- End-of-chapter links to questions in the accompanying Student Workbook have been updated to reflect the increased number of questions in the workbook.
- Student Workbook for Mathematics Explained for Primary Teachers.



Available separately, the *Student Workbook* for *Mathematics Explained for Primary Teachers*, 7th edition (ISBN: 978-1-5296-2631-5) is the perfect companion to your study. It contains 900 questions with detailed solutions and explanatory notes.

- Test your knowledge of key concepts and principles.
- Apply mathematical skills and knowledge in real-life situations and to mathematical problems and investigations.
- Consider how to respond to children's errors and misunderstandings and how to evaluate different teaching approaches.
- Be inspired to develop classroom ideas that promote understanding and mastery.

INTRODUCTION

This seventh edition has been revised to ensure that the content is in line with the mathematics programmes of study for Key Stages 1 and 2 (children aged 5 to 11 years) in England (DfE, 2013) and consistent with the current emphasis on teaching for mastery, promoted by the National Council for Excellence in Teaching Mathematics. As well as ensuring that I address all the statutory requirements for mathematics in primary schools in England that have to teach the National Curriculum, I am aware that many readers of this book will teach mathematics to this age range of children in other countries with a variety of statutory curriculums and also that there are many schools in England that are not required to teach the English National Curriculum. It has been encouraging to hear that teachers, teacher trainees and mathematics educators in many other countries have found the previous editions to be helpful and relevant to their work. I am confident that this will continue to be the case. In particular, I have ensured that the primary national curriculums of the other countries in the United Kingdom are covered comprehensively.

I have continued in my commitment to focus on what has always been the key message of 'Mathematics Explained': the need for priority to be given in initial teacher training and professional development to primary school teachers developing secure and comprehensive subject knowledge in mathematics, characterized by understanding and awareness of the pedagogical implications. Even well-qualified graduates feel insecure and uncertain about much of the mathematics they have to teach, as is demonstrated in Chapter 1 of the book. I know from experience just how much they appreciate a systematic explanation of even the most elementary mathematical concepts and procedures of the primary curriculum. In my long career in teacher education, I have often reflected on what qualities make a good teacher. I have a little list. Top of the list is the following conviction: the best teachers have a secure personal understanding of the structure and principles of what they are teaching. This book is written to help primary teachers, present and future, to achieve this in mathematics. It sets out to explain the subject to primary school teachers, so that they in turn will have the confidence to provide appropriate, systematic and careful explanation of mathematical ideas and procedures to their pupils, with an emphasis on the development of understanding, rather than mere learning by rote. This is always done from the perspective of how children learn, understand and develop mastery of this subject. Implications for learning and teaching are embedded in the text and highlighted as 'Learning and teaching points' distributed throughout each chapter.

Section A (Chapters 1–3) of this book is about mathematical understanding. Chapter 1, drawing on my research with trainee teachers, provides evidence for the need to develop understanding of mathematics and to lower anxiety about this subject in those who are to teach in primary schools. Chapter 2 considers the distinctive contribution that mathematics makes to the primary curriculum; and Chapter 3 – which I consider to be the key chapter in this book – is about pupils learning to learn mathematics with understanding.

Section B explains the key features of mathematical reasoning and problem solving and seeks to give these two major themes in the mathematics curriculum the prominence and priority they warrant.

Sections C–H then focus on the content and principles of various sections of primary school mathematics, covering all you need to understand about: numbers, including the number system and various number properties; calculations, mental and written, including those with fractions, decimals and percentages; algebra; the principles and concepts of measurement and units of measurement; geometry; and statistics and probability.

It is important that those who teach mathematics to children know and understand more mathematics than the children may have to learn. This will help the teacher to feel confident and to teach with authority in this area of the curriculum. It will also help them to be more aware of the significance of what the children are learning. So, in places in the book there will be material that may go beyond the immediate requirements of the primary school curriculum. In particular, some sections – such as some calculations with decimals and percentages and much of the material in Chapter 27 – are specifically provided for the personal development of the primary teacher, to help them to handle, with confidence, some of the encounters with mathematics they will have in their professional role.

Finally: a comment about calculators. Although calculators have effectively disappeared from most mathematics lessons in primary schools that are constrained by the National Curriculum in England, they are still available in primary schools elsewhere in the UK and in countries overseas. For their benefit, and particularly for the benefit of the readers of this book, I have included here and there a number of ways in which simple four-function calculators can be used effectively to provide insights and to promote understanding in learning mathematics. Readers should find that their engagement with these examples enhances their own understanding of the mathematical ideas being explained.

Derek Haylock, Norwich

SECTION A

MATHEMATICAL UNDERSTANDING

WATCH THE SECTION OPENER VIDEO AT: HTTPS://STUDY.SAGEPUB.COM/HAYLOCK7E

1

PRIMARY TEACHERS' INSECURITY ABOUT MATHEMATICS

IN THIS CHAPTER, THERE ARE EXPLANATIONS OF

- the importance of primary school teachers really understanding the mathematics they teach and being able to explain it clearly to the children they teach;
- the relationship of mathematics anxiety to avoidance of mathematical demand, rote learning and low levels of creativity in problem solving;
- attitudes of adults in general toward mathematics;
- mathematics anxiety in primary school teachers;
- the insecurity about mathematics of many primary trainee teachers.

READ THIS CHAPTER'S CURRICULUM LINKS AT: HTTPS://STUDY.SAGEPUB.COM/HAYLOCK7E

UNDERSTANDING AND EXPLAINING

Being a successful learner in mathematics involves constructing *understanding* through exploration, problem solving, discussion and practical experience – and also through interaction with a teacher who has a clear grasp of the underlying structure of the mathematics being learnt. For children to enjoy learning mathematics, it is essential that they should understand it; and that they should make sense of what they are doing in the subject, and not just learn to reproduce learnt procedures and recipes that are low in meaningfulness and purposefulness.

One of the ways for children to learn and understand much of the mathematics in the primary school curriculum is for a teacher who understands it to explain it to them. Those who teach mathematics in primary schools should ensure that the approach they take to organizing children's activities allows sufficient opportunities for them to provide teaching that includes engaging with children in question and answer, discussion and explanation – all aimed at promoting understanding and confidence in mathematics. Of course, there is more to learning mathematics than just a teacher explaining something and then following this up with exercises. The key processes of mathematical reasoning, applying mathematics and problem solving must always be at the heart of learning the subject – and these figure prominently in this book, particularly in Section B (Chapters 4 and 5).

But children do need 'explanation' to help them to understand mathematics, to make sense of it, to give them confidence and to help them have positive attitudes to the subject. There is now in England a greater awareness that primary teachers must organize their lessons and the children's activities in ways that give opportunities for them to provide careful, systematic and appropriate explanation of mathematical concepts, procedures and principles to groups of children. That many primary teachers have in the past neglected this aspect of teaching may possibly have been associated with a prevailing primary ethos which perhaps over-emphasized active learning and children discovering things for themselves. But it seems to me to be often a consequence of the teacher's own insecurity and anxiety about mathematics, which are characteristics of too many primary school teachers. This book is written to equip teachers with the knowledge, understanding and confidence they require to be able to explain mathematical ideas to the children they teach. English primary schools, following the example of some higher-performing countries in terms of mathematics achievement, are now putting an emphasis on teaching for what is termed 'mastery' in mathematics (see Chapter 3). Mastery approaches in teaching mathematics aim for a deep understanding of mathematical procedures, concepts and principles for all children. It is self-evident that a prerequisite for teaching for mastery is that primary school teachers themselves have this deep and secure understanding of the mathematics in the

primary school curriculum. Helping teachers and teacher trainees to feel confident in their own understanding and mastery of this subject is the principal purpose of this book.

ATTITUDES TO MATHEMATICS IN ADULTS

There are widespread confusions amongst the adult population in Britain about many of the basic mathematical processes of everyday life. This lack of confidence in basic mathematics appears to be related to the anxiety about mathematics and feelings of inadequacy in this subject that are common amongst the adult population. These phenomena have been demonstrated by surveys of adults' attitudes to mathematics (for example, Coben et al., 2003). Findings indicate that many adults, in relation to mathematical tasks, admit to feelings of anxiety, helplessness, fear, dislike and even guilt. The feeling of guilt is particularly marked amongst those with high academic qualifications, who feel that they ought to be more confident in their understanding of this subject. There is a perception that there are proper ways of doing mathematics and that the subject is characterized by questions to which your answers are either right or wrong. Feelings of failure, frustration and anxiety are identified by many adults as having their roots in unsympathetic attitudes of teachers and the expectations of parents. A project at King's College, London, looking at the attitudes of adults attending numeracy classes, found that the majority of such adults viewed themselves as failures and carried various types of emotional baggage from their schooldays. They spoke of their poor experience of schooling and of feeling that they had been written off by their mathematics teachers, usually at an early stage. Their return to the mathematics classroom as adults was accompanied by feelings of anxiety, even fear (Swain, 2004). Significantly, in a survey of over 500 adults in the UK, Lim (2002) identified three widely claimed myths about mathematics: it is a difficult subject; it is only for clever people (see also Chestnut et al., 2018); and it is a male domain.

MATHEMATICS ANXIETY IN TEACHERS AND LEARNERS

Research over many years into primary school teachers' attitudes to mathematics reveals that many primary teachers experience the same kinds of feelings of panic and anxiety when faced with unfamiliar mathematical tasks (Briggs, 1993), that they are muddled in their thinking about many of the basic mathematical concepts which underpin the

material they teach to children, and that they are all too aware of their personal inadequacies in mathematics. The widespread view that mathematics is a difficult subject, and therefore only for clever people, increases these feelings of inadequacy – and the common perception that mathematics is a male domain exacerbates the problem within a subset of the teaching profession that continues to be largely populated by women. The importance of tackling these attitudes to the subject was underlined by the findings of Burnett and Wichman (1997) that primary teachers' (and parents') own anxieties about mathematics can often be passed on to the children they teach. Witt and Mansergh (2008) identify the need to break this anxiety spiral in the course of initial teacher training.

It is important not to generate mathematics anxiety in the children we teach, because high levels of anxiety affect a person's ability to perform to their potential. Boaler (2022) argues that an unhelpful level of anxiety towards mathematics often starts early in the primary years of education and is found even amongst children who otherwise do well at school. She reports the well-established finding that a high level of mathematics anxiety leads to difficulties in learning the subject and avoidance of mathematical demand, and identifies in particular the way in which timed tests in mathematics contribute significantly to anxiety. The research of Ashcraft and Kirk (2001) and Ashcraft and Moore (2009), for example, confirms that raising anxiety about mathematics produces a drop in performance in the subject, particularly in terms of the individual's access to their 'working memory' and a tendency to avoid any kind of mathematical demand. Puteh (1998) has provided a helpful diagrammatic summary of research in this area (see Figure 1.1).

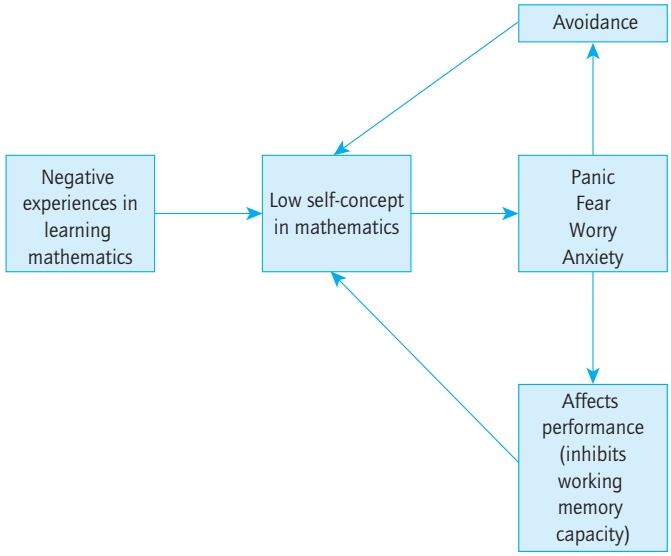


Figure 1.1 Some factors associated with mathematics anxiety
Source: Puteh (1998)

Newstead (1998) found that a teaching method that promoted understanding rather than just the memorization and rehearsal of procedures and recipes was associated significantly with lower levels of mathematics anxiety in primary school children. Khoule et al. (2017) report that the procedural method of teaching mathematics, when applied alone, is 'easy to forget or hard to remember' and is often associated with 'pain and frustration' for learners. Ford et al. (2005), also report that high anxiety towards mathematics has a negative effect on learners' performance. In particular, anxiety leads to reliance on learning mathematics by rote, rather than aiming and expecting to learn with understanding; this is a vicious circle, because reliance on rote memorization of rules rather than understanding them increases anxiety when faced with anything unfamiliar (Figure 1.2). Carey et al. (2016) provide a further analysis of the ways in which mathematics anxiety and mathematics performance are related.

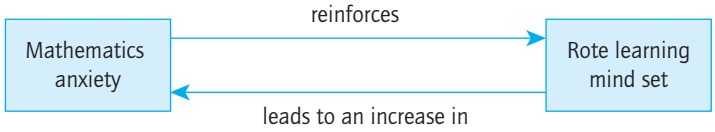


Figure 1.2 Mathematics anxiety and rote learning

In my own research I have found that anxiety about mathematics – which is reinforced when children are not being helped to learn mathematics in a meaningful way – is associated with rigid and inflexible thinking in unfamiliar mathematical tasks and leads to insecurity and caution when faced with a non-routine mathematical challenge, and therefore to low levels of creativity in problem solving (Figure 1.3). Creativity in mathematics is explained further in Chapter 4.

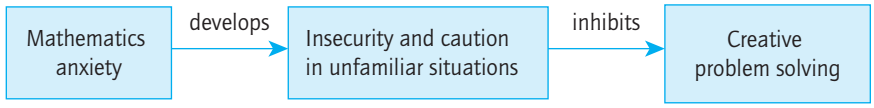


Figure 1.3 Mathematics anxiety inhibits creativity in problem solving

TRAINEE TEACHERS' ANXIETIES

The background for this book is mainly my experience of working with graduates enrolled on a one-year primary initial teacher-training programme. The trainee teachers I have worked with have been highly motivated, good honours graduates, with the

subjects of their degree studies ranging across the curriculum. Over a number of years of working with such trainees, it became clear to me that many of them start their course with a high degree of anxiety about having to teach mathematics.

An invitation was given for any trainees who felt particularly worried about mathematics to join a group who would meet for an hour a week throughout their course, to discuss their anxieties and to identify which aspects of the National Curriculum for mathematics appropriate to the age range they would be teaching gave them most concern. A surprisingly large number of trainees turned up for these sessions. Discussions with them revealed both those aspects of mathematics anxiety that they still carry around with them, derived clearly from their own experiences of learning mathematics at school, and the anxiety related to specific areas of mathematics they will have to teach where they have doubts about their own understanding.

Below, I recount some of the statements made by the primary trainee teachers in my group about their attitudes towards and experiences of learning mathematics. In reading these comments, it is important to remember that these are students who have come through the system with relative success in mathematics: all had GCSE grade C, or the equivalent. Yet this is clearly not how they feel about themselves in relation to this subject. The trainees' comments on their feelings about mathematics can be categorized under five headings: (1) feelings of anxiety and fear; (2) expectations; (3) teaching and learning styles; (4) the image of mathematics; and (5) language. These categories reflect closely the findings of other studies of the responses towards mathematics of adults in general and primary teachers in particular.

Feelings of Anxiety and Fear

When these trainee teachers talked freely about their memories of mathematics at school, their comments were sprinkled liberally with such words as 'frightened', 'terror' and 'horrific', and several recalled having nightmares. These memories were very vivid and still lingered in their attitudes to the subject as academically successful adults:

> Maths struck terror in my heart: a real fear that has stayed with me from over 20 years ago.
>
> I had nightmares about maths: I really did, I'm not joking. Numbers and figures would go flashing through my head. Times tables, for example. I especially had nightmares about maths tests.
>
> It worried me a great deal. Maths lessons were horrific.

Others recalled feelings of stupidity or frustration at being faced with mathematical tasks:

> I remember that I would always feel stupid. I felt sure that everyone else understood.
>
> Things used to get hazy and frustrating when I was stuck on a question.

Those of us who teach mathematics must pause and wonder what it is that we do to children that produces successful, intelligent adults who continue to feel like this about the subject.

Expectations

It seems as though the sources of anxiety for some trainee teachers were the expectations of others:

> It was made worse because Dad's best subject was maths.
>
> My teacher gave me the impression that she thought I was bad at maths. So that's how I was labelled in my mind. When I got my GCE result she said, 'I never thought you'd get an A!' So I thought it must just be a fluke. I still thought I was no good at maths.

But the most common experience cited by these trainees was the teacher's expectation that they *should* be able to deal successfully with all the mathematical tasks they were given. They recalled clearly the negative effect on them of the teacher's response to their failure to understand:

> There were few maths teachers who could grasp the idea of people not being mathematical.
>
> Teachers expect you to be good at maths if you're good at other things. They look at your other subjects and just can't understand why you can't do maths. They say to you, 'You should be able to do this …'.

Teaching and Learning Styles

The trainee teachers spoke with considerable vigour about their memories of the way mathematics was taught to them, recognizing now, from their adult perspective, that part of the problem was a significant limitation in the teaching style to which they were subjected:

> Surely not everyone can be bad at maths. Is it just that it's really badly taught?
>
> I remember one teacher who was good because she actually tried to explain things to me.

> My primary school teachers tried to help me to make sense of maths, but in secondary school maths teachers just taught us formal written methods without any understanding.

It was clear that most of the trainees in this group felt that they had been encouraged to learn by rote, to learn rules and recipes without understanding, particularly in secondary school. This rote-learning style (see glossary in Chapter 3) was sometimes reinforced by apparent success:

> I was quite good at maths at school but I'm frightened of going back to teach it because I think I've probably forgotten most of what we learnt. I have a feeling that all I learnt was just memorized by rote and now it's all gone.
>
> I could rote learn things, but not understand them.
>
> I got through the exams by simply learning the rules. I would just look for clues in the question and find the appropriate process.

The limitations of this rote-learning syndrome were sometimes apparent to the trainees:

> I found you could do simple problems using the recipes, but then they'd throw in a question that was more complex. Then when the recipe I'd learnt didn't work I became angry.
>
> We would be given a real-life situation but I would find it difficult to separate the maths concepts out of it.

But it seems that some teachers positively discouraged a more appropriate learning style:

> I was made to feel like I was a nuisance for trying to understand.
>
> Lots of questions were going round in my head but I was too scared to ask them.
>
> I always tried to avoid asking questions in maths lessons because you were made to feel so stupid if you got it wrong. There must be ways of convincing a child it doesn't matter if they get a question wrong.

The following remark by one trainee highlights how the role of trainee teacher serves to focus the feeling of anxiety and inadequacy arising from the rote-learning strategy adopted in the past:

> I have a real fear of teaching young children how to do things in maths as I just learnt rules and recipes. I have this dread of having to explain why we do something.

Image of Mathematics

For some trainees, mathematics had an image of being a difficult subject, so much so that it was acceptable to admit that you are not any good at it:

> Maths has an image of being hard. You pick this idea up from friends, parents and even teachers.
>
> My Mum would tell me not to worry, saying, 'It's alright, we're all hopeless at maths!' It was as if it was socially acceptable to be bad at maths.
>
> Among my friends and family it was OK to be bad at maths, but it's not acceptable in society or employment.

For some, the problem seemed to lie with the feeling that mathematics was different from other subjects in school because the tasks given in mathematics are seen as essentially convergent and uncreative:

> Maths is not to do with the creativity of the individual, so you feel more restricted. All the time you think you've just got to get the right answer. And there is only one right answer.
>
> There's more scope for failure with maths. It's very obvious when you've failed, because things are either right or wrong, so you feel a fool, or look a fool in front of the others.

Language

A major problem for all the trainee teachers was that mathematical language seemed to be too technical, too specific to the subject and not reinforced through their language use in everyday life:

> I find the language of maths difficult, but the handling of numbers is fine.
>
> Most of the words you use in maths you never use in everyday conversation.
>
> Some words seem to have different meanings in maths, so you get confused.
>
> I was always worried about saying the wrong things in maths lessons, because maths language seems to be so precise. I worry now that I'll say things wrong to children in school and get them confused. You know, like, 'Which is the bigger half?'

When we discussed the actual content of the National Curriculum programmes of study for mathematics, it became clear that many of the trainees' anxieties were related

to language. Often, they would not recognize mathematical ideas that they actually understood quite well, because they appeared in the National Curriculum in formal mathematical language, which they had either never known or forgotten through neglect. This seemed to be partly because most of this technical mathematical language is not used in normal everyday adult conversation, even amongst intelligent graduates:

> I can't remember what prime numbers are. Why are they called prime numbers anyway?
>
> Is a product when you multiply two numbers together?
>
> What's the difference between mass and weight?
>
> What is congruence? An integer? Discrete data? A measure of spread? A quadrant? An inverse? Reflective symmetry? A translation? A transformation?

Even as a 'mathematician', I must confess that it is rare for this kind of technical language to come into my everyday conversation, apart from when I am actually 'doing mathematics'. When this technical language was explained to the students, typical reactions would be:

> Oh, is that what they mean? Why don't they say so, then?
>
> Why do they have to dress it up in such complicated language?

It is clear then that unfamiliar terminology in mathematics inevitably increases anxiety for trainees. This is particularly significant given the current emphasis in teacher-training courses in Britain on 'teaching for mastery' in line with methods used in mathematics teaching in Singapore. This has brought with it new terminology, with which many trainees may be unfamiliar: for example, addend, subtrahend, bar-modelling, manipulatives.

MATHEMATICS EXPLAINED

Recognizing that amongst primary trainee teachers and, indeed, amongst many primary school teachers in general, there is this background of anxiety and confusion, it has always been clear to me that a major task for initial and in-service training is the promotion of positive attitudes towards teaching mathematics in this age range. The evidence from my conversations with trainee teachers suggests that to achieve this we need to shift perceptions of teaching mathematics away from the notion of teaching

recipes and more towards the development of understanding. And we need to give time to explaining mathematical ideas, to the ironing out of confusions over the content and, particularly, to the language of mathematics. Some trainees' comments later in the year highlighted the significance to them of having mathematics explained. The emphasis on explaining and understanding paid off in shifts of attitudes towards the subject:

> It's the first time anyone has actually explained things in maths to me. I feel a lot happier about going into the classroom now.
>
> The course seems to have reawakened an interest in mathematics for me and exploded the myth that maths was something I had to learn by rote for exams, rather than understand.
>
> I was really fearful about having to teach maths. That fear has now declined. I feel more confident and more informed about teaching maths now.

These kinds of reactions prompted me to write this book. By focusing specifically on explaining the language and content of the mathematics that we teach in the primary age range, this book will help trainee teachers – and primary school teachers in general – to develop this kind of confidence in approaching their teaching of this key subject in the curriculum to children who are at such an important stage in their educational development.

RESEARCH FOCUS: SUBJECT KNOWLEDGE FOR TEACHING

In the context of increasing government concern about the subject knowledge in mathematics of trainee teachers, a group of mathematics educators at the London Institute of Education audited trainee primary trainees' performance in a number of basic mathematical topics. Those topics in which they had the lowest facility were: making algebraic generalizations; Pythagoras's theorem; calculation of area; mathematical reasoning; scale factors; and percentage increase. Significantly, trainees with poor subject knowledge in mathematics were found to perform poorly in their teaching of mathematics in the classroom when assessed at the end of their training (Rowland et al., 2000). Further research in this area (Goulding et al., 2002), which included an audit of subject knowledge of primary teacher trainees and its relationship with classroom teaching, identified weaknesses in mathematical understanding, particularly in

the syntactic elements of mathematics, and a link between insecure subject knowledge and poor planning and teaching. Hourigan and O'Donoghue (2013) report similar findings in relation to elementary teachers in Ireland, identifying particular difficulties in knowledge of rational numbers (fractions), conceptual understanding and problem solving in mathematics. Rowland and Ruthven (2011: 1) argue that 'the quality of primary and secondary school mathematics teaching depends crucially on the subject-related knowledge that teachers are able to bring to bear on their work' – a conclusion based on their bringing together of the reflections and implications of a range of mathematics education researchers (see also the Research Focus for Chapter 27).